Gravimetric survey terrain correction using linear analytical approximation

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ABSTRACT
Various methods for computing the terrain correction in a high-precision gravity survey are currently available. The present paper suggests a new method that uses linear analytical terrain approximations. In this method, digital terrain models for the near-station topographic masses are obtained by vectorizing scan images of large-scaled topographic maps, and the terrain correction computation is carried out using a Fourier series approximation of discrete height values. Distant topography data are represented with the help of digital GTOPO30 and Shuttle Radar Topography Mission cartographic information. We formulate linear analytical approximations of terrain corrections for the whole region using harmonic functions as the basis of our computational algorithm. Stochastic modelling allows effective assessment of the accuracy of terrain correction computation. The Perm Krai case study has shown that our method makes full use of all the terrain data available from topographic maps and digital terrain models and delivers a digital terrain correction computed to a priori precision. Our computer methodology can be successfully applied for the terrain correction computation in different survey areas.

Key words: Gravity, Data processing, Computing aspects.

INTRODUCTION

Computation of the terrain corrections \( \delta g_{tc} \) is necessary for gravity survey data processing. The computation involves integration over the volume \( V \), bounded by the terrain surface and a “normal” surface (plane, sphere, or ellipsoid) (Hammer 1939), crossing the gravity station. When the “normal” surface is a horizontal plane \( z = \text{const} \) (the most important case in practice), the expression for the terrain correction value is as follows:

\[
\delta g_{tc}(x, y, z) = \gamma \sigma \int \int \int_{V} \frac{(\zeta - z)}{L^3} \ d\xi \ d\eta \ d\zeta ,
\]

where \( x, y, \) and \( z \) are the rectangular coordinates of the gravity station; \( \gamma \) is the gravitational constant; \( \sigma \) is the interbedded layer density; \( \xi, \eta, \) and \( \zeta \) are the coordinates of the considered mass element; and \( L = \sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2} \) is the distance between the mass element and the gravity station.

Various methods have been developed for \( \delta g_{tc} \) computation using master charts, nomograms, and computer methodologies (Jackson and van Gulik 1983; Zhou, Zhong, and Li 1990; Nowell 1999; Tsoulis, Wziontek, and Petrovic 2003).

In order to achieve high accuracy of \( \delta g_{tc} \) computation, the gravity forward modelling (1) requires an elevation grid with high resolution near the computational point and a coarser grid as the distance from the point increases. Therefore, the area of the terrain effect is conventionally divided into several non-crossing zones, each one being portrayed with a different level of detail (Cogbill 1990; Ma and Watts 1994; Banerjee 1998; Hinze et al. 2005). In Russia, they are usually called central, near, intermediate and far zones.

Achieving better accuracy of \( \delta g_{tc} \) computation is relevant to the current need for large-scale high-precision gravity
surveys in severe topographies. It is impossible to achieve optimal geological efficiency of gravity surveys, even using the latest equipment (land gravity meter for microlag accuracy, etc.), without thorough consideration of all factors used in the overall reduction of the gravitational field. The most important of these factors is the effect of the terrain (Leaman 1998).

There is therefore an urgent need for \( \delta g_t \) computation that takes advantage of developments in the theory of geopotential field interpretation and of enhanced computing capabilities to deal with bulk data furnishing high-level terrain detail. Clearly, such algorithm and software development should also be accompanied by the development of methods for the preparation of initial data (Blais and Ferland 1984; Hećimović and Bašić 2005).

To compute terrain corrections in gravity survey with precision, as far as possible, compatible with initial quality of the data, we suggest applying linear analytical approximations of heights and values of corrections. The area of the terrain effect \( D \) is divided into two zones: an inner zone \( D_1 \), which includes central and near zones, and an outer zone \( D_2 \), including intermediate and far zones. For these zones, different datasets and algorithms for \( \delta g_t \) computation are used. For zone \( D_1 \) we suggest using linear analytical approximations of heights, and for \( D_2 \), analytical approximations of terrain correction values are made. This makes full use of all terrain data.

Our approach also allows simulation of any topographic features except for mountainous areas with a slope angle above 70°, for which cases, gravity ground surveys are not usually conducted. Our computations do not take into account the Earth’s sphericity as local gravimetric surveys in Russia conventionally apply the Bouguer plate. However, accounting for the sphericity in analytical terrain models (ATMs) involves no difficulty and is theoretically explored by Strakhov (2007).

**A NEW APPROACH TO TERRAIN CORRECTION**

The main characteristic of our method for terrain correction computation is the use of appropriate approximation algorithms. For the \( D_1 \) zone, covering central and near regions, we use, on grounds of efficiency, linear analytical approximations of the terrain \( z = \Psi(x, y) \), as developed by V.N. Strakhov (Strakhov 2007) and his students (Dolgal 1999; Stepanova 2007; Kerimov 2009). The ATM represents all topographic features and is used instead of an array of heights of the discrete terrain model.

Digital models of the “local” terrain in zone \( D_1 \) are obtained by vectorization of scanned images of large-scale topographic maps. This yields better accuracy of the terrain data and, consequently, of the terrain correction computation, as compared with the traditional technologies based on hand coding of topographic maps at the regular grid nodes. The area of zone \( D_1 \) can range from a few to several hundred square kilometers, depending on the desired accuracy of the terrain corrections. Increasing or decreasing the area of zone \( D_1 \) results in different detail levels of the digital terrain model (DTM) obtained from large-scale topographic maps and thus influences the accuracy of terrain correction.

Using analytical terrain approximations allows, firstly, a more accurate representation of the terrain in comparison with rectangular parallelepiped approximation; secondly, height correction computation at gravity stations themselves to give an array of terrain heights around each station; thirdly, significant data reduction at calculations due to use of a set of functional dependences instead of a file of discrete heights; and fourthly, height calculation at nodes of an arbitrary grid, choosing the parallelepiped size and the \( D_1 \) zone radius depending on the desired computation accuracy.

For zone \( D_2 \) it is more efficient to carry out sourcewise approximation of \( \delta g_t \) values, previously determined for nodes of a comparatively wide-spaced regular grid. The next stage is 3D interpolation of terrain corrections directly to the gravity stations by gravimetric forward modelling. By sourcewise approximation, we mean selecting elementary source parameters yielding a modelled field closely identical to the field \( \delta g_t \). The area of zone \( D_2 \) can be from tens of thousands to a few hundred thousand square kilometers. Special features of the “regional” terrain can be represented with a high level of detail using data in the GTOPO30 height matrix and Shuttle Radar Topography Mission (SRTM) reference tool (Dolgal, Bychkov, and Antipin 2003; Hećimović and Bašić 2005; Jia, Davis, and Groom 2009; Tsoulis, Novák, and Kadlec 2009), covering almost the entire surface of the Earth and freely available on the Internet. GTOPO30 is a 30-second DTM obtained by the US Geological Survey, and SRTM is a 3-second DTM obtained by satellite radar interferometry survey (National Imagery and Mapping Agency, USA). The division of a survey area into zones \( D_1 \) and \( D_2 \) provides high-speed calculation of corrections \( \delta g_t \) without loss of accuracy. On the one hand, an ATM for zone \( D_1 \) yields more detailed terrain data, and on the other hand, the use of analytical terrain approximations for zone \( D_2 \), where a less accurate DTM can be used, allows less computing time.
**Figure 1** Reference terrain data (a: terrain; b: map fragment): (1) map fragment contour; (2) gravimetric stations; (3) height points of the vectorized topographical map; (4) nodes of the SRTM height matrix; and (5) nodes of the GTOPO30 height matrix.

**DATABASE ASSESSMENT**

Efficient preparation of DTMs can be achieved by scanning topographic maps and vectorizing scanned images with custom programs. This allows fast generation and storage of large datasets that digitally codify features of large- to medium-sized topographic maps. Vectorization of the terrain depth maps results in the creation of database files (for example: *.shp files or *.dat files in ASCII), containing $10^5$–$10^7$ vectors $\mathbf{v} = \{x, y, z\}$ with a typical height density grid of 200–300 points/cm$^2$ in the map scale (Fig. 1). Figure 1b shows a case with 19 gravity stations, about 600 height points of the vectorized topographic map, 324 nodes of the SRTM DTM height matrix, and 4 nodes of the GTOPO30 matrix. The coordinates and heights of gravity stations were determined with the help of GPS Trimble 5700 and from tachometers. The standard error for the determination of the station coordinates is $\pm 5$ m and $\pm 7$ cm for their heights.

Another important issue, closely connected with improving the accuracy of the terrain correction, is the difference in height of cartographic values (as represented on topographic maps) and topographic values (instrumentally measured for gravimetric survey stations). Analysis of differences of heights $\Delta z$, obtained from vectorizing differently scaled topographic maps and from gravity survey measurements, revealed that the overall distribution of height differences for different regions of Russia is close to Gaussian normal distribution. The root-mean-square (RMS) deviation of height differences $\Delta z$ cannot be entirely explained by errors of gravity station horizontal positioning. Differences in the $\Delta z$ value are mainly due to the cartographic error $\delta_2$ (Fig. 2b).

It can be assumed that the differences obtained in values $\Delta z$ are due to two factors: errors $\delta_1$ of horizontal positioning of the gravity stations and mistakes $\delta_2$ caused by errors of height values $z$, presented on topographic maps themselves. Assuming both factors to be independent of each other, we can use variance analysis to determine numerical values of $\delta_2$: $D(\Delta z) = D(\delta_1) + D(\delta_2)$, where $D$ is a symbol for dispersion. In order to determine the contribution of the $\delta_1$ component to the $\Delta z$ value, stochastic simulation was performed to assess its amplitude. According to the results of the analysis (Fig. 2), the differences between topographic and cartographic heights $\Delta z$ are mainly due to the cartographic error $\delta_2$ (Fig. 2b).

To assess the accuracy of heights in the GTOPO30 and SRTM matrices and their possible use in terrain correction computation for zone $D_2$, we compared the DTM heights with those measured instrumentally during gravity surveys in different regions of Russia. The value of differences primarily depends on the terrain roughness parameter (Table 1). The dependence of the RMS error of heights on the average terrain roughness is almost linear. Height values in the SRTM matrix are 5–10 times more accurate than those in GTOPO30. It should be noted that the SRTM heights are usually 5–10 m higher than those instrumentally measured due to the area forest coverage. The more accurate SRTM matrix allows for a significant reduction of the far zone inner radius to 1–5 km, depending on the extent of the area forest coverage. The histograms for differences between topographic heights and those in the GTOPO30 and SRTM matrices (Fig. 3)
show that the statistical distribution of height deviations is close to normal.

THE CENTRAL ZONE PROBLEM

Special attention should be paid to digital elevation models for central and near-station terrain correction $\delta_{g_{tc}}$ as the greatest effects in the gravity field come from these zones (Schiavone, Capolongo, and Loddo 2007, 2009). The contribution of the distant zone effects is much less. In rough topography, the gradient of the central zone correction can reach $5 \, \text{E}^{-2}$; therefore, the level of correction accuracy is very important to determine small-amplitude anomalies. In most cases, the “star” method, using data obtained by leveling along the rays around gravity stations, is applied, but it can be time and effort consuming for rough topographies with a forest cover. Moreover, in mountain taigas, leveling around each station is almost impossible.

A very promising method for the construction of digital elevation models of the near-station zone is laser scanning (Schiavone et al. 2007), which provides highly accurate modelling of the irregular topographic surface and seems highly productive in comparison with other methods. However, laser scanning always includes the need for an unobstructed field of view.
Table 1 Comparison of the GTOPO30 and SRTM matrices representing terrain heights instrumentally measured in gravity surveys for different areas.

<table>
<thead>
<tr>
<th>№ of area</th>
<th>Number of stations</th>
<th>Terrain characteristics</th>
<th>Differences of heights, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Heights, m</td>
<td>Roughness, m/km</td>
</tr>
<tr>
<td></td>
<td></td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>1</td>
<td>2720</td>
<td>137.55</td>
<td>520.07</td>
</tr>
<tr>
<td>2</td>
<td>2338</td>
<td>110.91</td>
<td>459.94</td>
</tr>
<tr>
<td>3</td>
<td>6295</td>
<td>107.21</td>
<td>217.98</td>
</tr>
<tr>
<td>4</td>
<td>3242</td>
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</tr>
<tr>
<td>5</td>
<td>1573</td>
<td>161.26</td>
<td>359.34</td>
</tr>
<tr>
<td>6</td>
<td>3631</td>
<td>123.70</td>
<td>246.32</td>
</tr>
<tr>
<td>7</td>
<td>2604</td>
<td>142.05</td>
<td>502.90</td>
</tr>
<tr>
<td>8</td>
<td>3268</td>
<td>89.40</td>
<td>394.69</td>
</tr>
<tr>
<td>9</td>
<td>3159</td>
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<td>108.77</td>
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<tr>
<td>14</td>
<td>2451</td>
<td>116.03</td>
<td>389.06</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Terrain roughness was calculated as the relation of height differences of gravimetric stations, located on profiles, to station spacing making of 100–250 m.

In order to calculate errors for the central zone correction under different terrain models, a detailed tacheometric survey was carried out on three test sites of a detailed gravity work area in Perm Krai (Fig. 4). The sites differed in terrain morphology and roughness and in forest coverage. Horizontal and altitudinal positions of the test site topographic irregularities were recorded with the help of pickets at 5- to 10-m spacing. The accuracy of the picket coordinates and heights was ±0.02 m.

The work resulted in various DTMs with different resolutions, coming from the following data sources:

- detailed tacheometric survey on the test sites;
- vectorizing 1:5 000, 1:25 000, and 1:50 000 topographic maps;
- SRTM.

The analysis of topographic and cartographic height differences revealed that height errors on 1:5 000, 1:25 000, and 1:50 000 topographic maps correspond to ±1.0-, 3.5-, and 7.0-m RMS errors, respectively.

The obtained DTMs allowed the correction computation for the central zone with an area of 100 m×100 m. The computation was carried out using ATMs according to the following methodology. For the database, we used correction values of the DTMs extracted from the detailed tacheometric survey. The central zone corrections varied from 0.009 mGal to 0.224 mGal. The difference between terrain corrections and their precise values obtained from the detailed tacheometric survey are represented in Table 2.

Analysis of the central zone correction under different terrain models showed sufficient computation accuracy us-
ing different databases. Nevertheless, to correctly compute the correction, all sources of the terrain data, ranging from detailed gravity survey instrumental data to large-scale 1:5 000–1:25 000 topographic maps, should be used. The main reason for the central zone correction errors seems to be the digital elevation model low resolution obtained from topographic data.

Considering different precision values of topography data derived from different sources, it is inappropriate to mix different databases in one array. Analytical approximations should be applied to each database separately, determining the zone area in accordance with the data accuracy.

**“LOCAL” TERRAIN CORRECTION COMPUTATION**

There may be large differences between heights of gravity stations obtained from field works and from topographic maps. Combined use of these data can lead to big errors in the computation of terrain corrections. To compute terrain correction, we formulate linear analytical approximations for the topographic masses. In general, the ATM represents not discrete heights but a set of functionally dependent parameters and provides desirably precise dependence of an arbitrary point height \( z \) on the horizontal coordinates \( x, y \) of the point. The ATM helps minimize “instrumental” and “topographic” height differences by “projecting” an irregular grid \((x_p, y_p, z_p)\) for gravitational field calculation on the topography surface and optimize the computation process. The incontestable advantage of the ATM is the possibility to recover height values \( z \) for the arbitrary grid nodes.

Dolgal et al. (2003) experimentally determined that high-resolution topography can be expressed by a double Fourier series with a limited number of terms. For approximating functions \( z \approx \Psi(x, y) \), trigonometric functions are chosen. Heights \( z \) are then represented by a double Fourier series segment:

\[
z \approx \Psi(x, y) = \sum_{m=0}^{P} \sum_{n=0}^{Q} C_{mn} \exp \left( -2\pi i \left( \frac{mx}{L_x} + \frac{ny}{L_y} \right) \right),
\]

where \( L_x \) and \( L_y \) are the survey area linear dimensions along coordinate axes \( OX \) and \( OY \); \( C_{mn} \) are the Fourier coefficients, \( u = 0, 1, 2, \ldots, P; v = 0, 1, 2, \ldots, Q; \) and \( P, Q \) are integer boundary harmonics in the Fourier spectrum.

For calculating coefficients \( C_{uv} \), the fast Fourier transform is applied

\[
C_{uv} = \frac{1}{M} \sum_{m=0}^{M-1} \left( \frac{1}{N} \sum_{n=0}^{N-1} z_{mn} \exp \left( -im\frac{2\pi}{N} \right) \right) \exp \left( -imu\frac{2\pi}{M} \right),
\]

i.e., the two-dimensional discrete Fourier transform successively computes one-dimensional transforms for the \( N \) lines and then \( M \) columns of matrix \( z \).

Maximum approximation accuracy is known to be achieved at \( P_{\text{max}} = M/2, Q_{\text{max}} = N/2 \) (Kahaner, Moler, and Nash 1989). The matrix in Fig. 5a has 500 lines and 500 columns; \( P = Q = 250 \). The RMS error for approximation of the topography by the double Fourier series reached only \( \pm 0.16 \) m (Fig. 5b), with elevation change over 200 m. The information on a relief contains in a set of coefficients \( C_{uv} \) (Fig. 5c).

To optimize the computation process, the Fourier series is truncated (the required values \( P < P_{\text{max}} \) and \( Q < Q_{\text{max}} \) are selected). The number \( K_{\text{dis}} \) of discarded coefficients \( C_{uv} \) depends on the terrain roughness, as desired by the RMS height deviation \( \sigma \), and the desired computation accuracy \( \delta \) of the terrain corrections \( \delta g_{zuv} \). We have experimentally established that an expression for the generalized regression \( K_{\text{dis}} = \phi(\sigma, \delta) \) is as follows:

\[
K_{\text{dis}} = \exp[a + b\sigma + c\delta^{1.5}],
\]

where \( a = -0.69447333, b = 0.0083031678, \) and \( c = 0.0029217935 \) are coefficients calculated by the least squares method.

A graphical representation of this dependence is plotted in Fig. 6.

With regard to equation (4), we constructed an ATM (2) that allows creating an array of heights around each gravity station in matrix form, with the matrix’s dimension \( K \times K \) and step \( d = \Delta x = \Delta y \) that, as shown below, is optimized during
Gravimetric survey terrain correction

Figure 5 Terrain approximation by the double Fourier series: (a) reference terrain depth map; (b) terrain depth map obtained by inverse Fourier transformation; and (c) energy spectrum.

Figure 6 Dependence of correction accuracy on RMS of terrain relief and number $K_{d_{0}}$ of the discarded terms of the double Fourier series.

The terrain correction for the point $(x_{p}, y_{p}, z_{p})$ is calculated from the expression:

$$
\delta_{gtc}(x_{p}, y_{p}, z_{p}) = \sigma \sum_{i=1}^{K} \sum_{j=1}^{K} C_{ij}^{par},
$$

(5)

where $\sigma$ is the density, and $g_{ij}^{par}$ is the unitary rectangular parallelepiped gravitational effect at density 1.0 g/cm$^3$. Computation $g_{ij}^{par}$ is calculated by Rempel’s approximate formula (Rempel 1980):

$$
g_{ij}^{par} = \frac{\gamma d}{R} \left[ \sqrt{(z_{ij} - z_{p})^2 + (R_{ij} + 0.5d)^2} - \sqrt{(z_{ij} - z_{p})^2 + (R_{ij} - 0.5d)^2} + d \right],
$$

(6)

where $R_{ij} = \sqrt{(x_{i} - x_{p})^2 + (y_{j} - y_{p})^2 - 0.075d^2}$, $x_{i}$, $y_{j}$, $z_{ij}$ are the parallelepiped base centre coordinates, and $\gamma$ is the gravitational constant. Equation (6) provides high-accuracy computation if $R \geq d$ (Rempel 1980). It should be noted that, for $x = x_{p}$ and $y = y_{p}$, i.e., for the central parallelepiped, when $R$ is a complex value, the computation is not carried out as for $z = z_{p}$ the value of $g_{ij}^{par}$ is equal to zero. The computation time using formula (6) is much faster than for other methods used to accelerate the computation process (Jackson and van Gulik 1983; Parker 1995, 1996; Hwang, Wang, and Hsian 2003; Tsoulis et al. 2003).

The outer contour of Zone $D_1$ is a square with a gravity station $(x_{p}, y_{p}, z_{p})$ at its centre. Correction $\delta_{gtc}$ is computed with the help of an algorithm that adaptively chooses the elementary parallelepiped cell sizes to approximate the volume $V$ sufficiently closely so as to yield an error in the computation of formula (1) for $\delta_{gtc}$ that matches an a priori given error.

Zone $D_1$ itself is divided into progressively smaller sub-zones $G_{1} = G_{1}^{1} \cup G_{1}^{2} \cup G_{1}^{3}...$ with automatically chosen parallelepiped dimensions (Fig. 7). The level of accuracy of integrals (1) is estimated using adaptive automatic quadrature algorithm (Kahaner et al. 1989). Changing the parallelepiped dimensions $d$ in the process of computation is not found to be problematic as the analytical description of the topography (a set of the Fourier coefficients $C_{uv}$) allows for the reconstruction of zone $D_1$ of arbitrary point heights by trigonometric interpolation using expression (2). A similar approach is used, applying adaptive discretization by the quadtree method (Davis, Kass, and Li 2010).

The computation scheme for the “local” correction $\delta_{gtc}$ is as follows. The outer computation cycle is based on...
Figure 7 Example of the “local” terrain correction computation within the radius of 0–5 km. (a) Terrain relief \( H \). (b) The number of approximating parallelepipeds \( k \) per unit area. (c) The correction map \( \delta g_{\text{tc}} \).

ground stations \( K \). The number of inner cycles depends on
the topographic complexity, each inner cycle \( k = 1, 2, 3, \ldots \) requiring:
- making, with the help of approximating functions \( z \approx \Psi(x, y) \), a local array \( \{z^k\} \) for subzone \( \mathcal{G}_1 \) with points at \( \Delta^k \) spacing;
- gravity forward modelling using formula (5);
- assessing accuracy of the resultant value and exiting the
loop when the difference of the terrain corrections calcu-
lated at two different networks of heights does not exceed
an a priori set value (required computation accuracy of
 Corrections), or reducing the network of cell heights \( \{z^k\} : \Delta^{k+1} < \Delta^k \) (we take \( \Delta^{k+1} = \Delta^k/2 \)) and repeating the cycle.

In this way, correction \( \delta g_{\text{tc}} \) is computed for the whole \( D_1 \)
zone with an a priori given accuracy simultaneously achieved
for an optimal number \( K \) of elementary parallelepipeds that
approximate the volume \( V \). The number \( K \) increases with
severity of topography and decreases with progressive flatness
(Fig. 7b). The obtained terrain corrections \( \delta g_{\text{tc}} \) are represented
in Fig. 7c.

“REGIONAL” TERRAIN CORRECTION
COMPUTATION

The algorithm for “regional” terrain correction computation
in zone \( D_2 \) includes construction of the matrix of approxi-
mate analytical corrections (a brief description of sourcewise
approximation is given in the Appendix) and correction com-
putation at gravity stations by 3D interpolation. The analyt-
cal approximations are iteratively obtained, and as iteration
criterion for solving the system of linear algebraic equations
(SLAE), we use either the completion of a given number of
iterations or the attainment of a given degree of coincidence
\( \epsilon \) between the source \( \delta g_{\text{sc}} \) and model \( U \) field in the Euclidean
metric \( L^2 \) (Dolgal 1999; Dolgal and Pugin 2006). The correction
computation is preceded by verifying that each gravity
station has coordinates that belong to area \( S \), for which the
analytical approximation was built. If the gravity station is lo-

cated beyond \( S \), the resultant value of correction \( \delta g_{\text{tc}} \) is coded
as undefined, and the correction is computed by forward modelling
(equation (A2) in the Appendix).

Computation of distant zone correction using a “dense”
reference DTM causes additional computational costs. Con-
sequently, the number of approximating parallelepipeds in
such zones should be as small as possible to reach the desired
computation accuracy. Approximating the reference DTM by
the double Fourier series allows use of a coarser height grid,
thus roughening the terrain model for zone \( D_2 \). This leads
to significant computation time saving on the one hand and
allows the desired correction accuracy to be reached on the
other. In this case, the ATM of the whole region is used to
construct a terrain model for the survey area with respect to
functions (4).

THE PERM KRAI CASE

Consider the example of the “regional” terrain correction
computations for the points of a regular grid (resolution equal
to \( 1 \times 1 \) km) created from GTOP30, describing the Perm
Krai and adjacent areas within 200 km from the boundary of
the region. The total DTM consisted of 1812 lines and 1236
columns. The test area outer and inner radii were 200 km
and 20 km, respectively. Thus, corrections were calculated at
246,715 nodes of the grid (Fig. 8). The computer time on an
Intel Core2 Duo personal computer running at 2.666 GHz
was about 1.5 hours. The largest correction values (up to
4.6 mGal) were obtained for the northeast of the test area, where height values of the Ural Mountains can reach up to 1500 m. For the flat part of the area, distant zone terrain corrections did not exceed 0.05 mGal.

At the next stage, the extracted corrections were analytically approximated. The elementary source parameters were iteratively computed by solving the SLAE. The termination criterion was either that the requested number of iterations had been performed or the requested degree of the source and model field coincidence had been reached. After 22 iterations in a time of 20 minutes, an approximation structure of 246,715 point sources with an RMS error $\varepsilon$ of 0.0009 mGal was obtained.

The final stage of correction computations was the gravity field recovery by means of forward modelling of the approximation structure directly at gravity stations within the test area. The extracted analytical approximation can then be successfully used to compute the distant zone correction for other areas of gravity surveys in Perm Krai.

**ASSESSMENT OF THE TERRAIN CORRECTION COMPUTATION ACCURACY**

Perturbing factor effects can be estimated with sufficient objectivity via simulation calculations of the correction $\delta g_{tc}$ under real gravity survey conditions.

The simulation process can be exemplified by calculating the correction $\delta g_{tc}$ in a rectangular system of coordinates. The
expression for the correction $\delta g_{ic}$ of the gravitational field is as follows:

$$\delta g_{ic}(x, y, z) = \sum_{i=1}^{k} \sum_{j=1}^{k} \Omega(u_{ij}),$$

where $u = \{u_1, u_2, \ldots, u_m\}$ is the $m$-dimensional parameter vector, characterizing physical and geometrical parameters of a separate approximation cell and its location relative to the correction $\delta g_{ic}$, computation point with the coordinates $(x, y, z)$; $\Omega$ is the forward modelling operator; and $k \times k$ is the number of separate cells within zone $D$.

We shall further consider a vector $p = \{p_1, p_2, \ldots, p_m\}$ with a structure similar to vector $u$ but with one or more components perturbed by a random element $\varepsilon$, i.e., $p_n = u_n + \varepsilon$, where $n = 1, 2, \ldots, m$. The expression for the correction error $\Delta g$ at a particular point due to the random deviations in the database is as follows:

$$\Delta g(x, y, z) = \sum_{i=1}^{k} \sum_{j=1}^{k} \Omega(p_{ij}) - \sum_{i=1}^{k} \sum_{j=1}^{k} \Omega(u_{ij}).$$

Correction errors expressed by (8) are estimated using the Monte Carlo method. Modelling randomness is performed by generating a sequence of pseudorandom numbers $\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots$. Statistical characteristics of the sequence can be selected on the basis of a priori data on the geological environment and technical characteristics of the survey. Computation is made for a set of points in space, and from these, the statistical parameters and distribution law for $\Delta g$ are determined.

Here we give an example of assessing the terrain correction computation accuracy by stochastic modelling for an oil-perspective part of Western Ural (in the east of Perm Krai). On the test site of 50 km$^2$ in area, a 1:25 000 scaled gravity survey was carried out using a grid with a resolution of 500 m $\times$ 100 m. Observations were made at 1575 stations. The height differences within the test area varied from 160 m to 360 m. The correction errors were estimated for all gravity stations of the test area in two variants.

In the first variant, we modelled random space deviations of gravity stations on the plan from their real locations (Fig. 9). Deviations of each X and Y coordinate were assumed mutually independent; the deviation amplitudes were normally distributed with expectation mean $M = 0$ and standard deviations $S = 14.1$ m. Thus the maximum total deviation $\varepsilon_{xy}$ of the gravity station did not exceed, $\varepsilon_{xy} = \sqrt{14.1^2 + 14.1^2} \approx \pm 20$ m with a probability of $<68\%$ (at $M \pm S$).

Computation algorithms for the inner zone $D_1$ and outer zone $D_2$ involved only cartographic heights of gravity stations. Thus, generating gravity station deviations on the plan caused corresponding errors in their heights, i.e., correction $\delta g_{ic}$ errors due to horizontal and vertical deviations of the gravity stations were modelled.

In the second variant, randomly oriented space height deviations ($\Delta z$) of the reference DTM were generated, i.e., the source terrain matrix was perturbed by interferences $\varepsilon_h$. The amplitudes of the height deviations were normally distributed with zero mean ($M = 0$) and standard deviation of 5 m. The values of the height deviations in the DTM are determined from the height errors in the reference DTMs.

Terrain correction errors were found from the difference between corrections, calculated for the original locations of gravity stations from the reference DTM and for their $\varepsilon_h$ perturbed DTM. For the inner zone $D_1$ (a square with 300-m sides), these errors vary from $-0.055$ mGal to $0.056$ mGal (mean of $0.0001$ mGal and RMS of $\pm0.0075$ mGal). For the outer zone $D_2$ with outer and inner limiting squares of 5100-m and 300-m sides, respectively, these errors vary from $-0.061$ mGal to $0.068$ mGal (mean of $0.0001$ mGal and RMS of $\pm0.009$ mGal). The statistical distribution of errors follows the Gaussian law (Fig. 9). Thus, the maximum RMS error for the total terrain correction within the radius of 2550 m will not exceed $\pm0.017$ mGal.
CONCLUSION

Terrain correction computation is one of the central issues in precise gravity field investigations. We have presented a methodology for terrain correction computation in a gravity survey. We use state-of-the-art methods to prepare primary cartographic data and mathematical techniques such as linear analytical approximations of discrete functions to describe gravitational anomalies and the Earth’s topography.

We have suggested the division of the survey area $D$ into two non-crossing zones: an inner zone $D_1$ (“local” terrain) and an outer zone $D_2$ (“regional” terrain). Different computation algorithms are applied to obtain the correction $\delta g_{tc}$ in each zone. Terrain data for zones $D_1$ and $D_2$ can be obtained from vectorizing scanned images of large-scale topographic maps and from Internet resources (GTOP030 and SRTM) respectively.

For zone $D_1$, analytical approximations of topographic masses are constructed from double Fourier series using fast inversion. Adaptive cell discretization allowed computation of the correction $\delta g_{tc}$ with the requested accuracy. Simultaneously, the effect of the near-station (the central zone) topographic masses can be considered in the same computation cycle.

For zone $D_2$, we used a sourcewise approximation of the corrections calculated at nodes of a relatively coarse regular grid and then computed the correction $\delta g_{tc}$ by forward modelling of the approximation structure with known parameters, i.e., we performed a 3D interpolation of corrections. The obtained analytical approximation for the whole region can be then successfully applied for the “regional” terrain correction computation in different survey areas, as exemplified by the Perm Krai (Western Ural) case study.

We have shown that our computer methodology for terrain correction calculations in gravity prospecting is an effective means of assessing the correction accuracy using digital cartographic and topographic data, analytical terrain approximations, and stochastic simulation.

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**APPENDIX**

It is known that the function $\delta g_{sc}(1)$ is harmonic, satisfying the Laplace equation. Harmonic functions have the uniqueness property: two harmonic functions matching on a closed surface are the same everywhere inside it. The harmonic function is completely determined by its values on closed surfaces. Consequently, if we can find the auxiliary harmonic function $U$, satisfying the condition:

$$\|\delta g_{sc} - U\|_{L_2} \leq \varepsilon,$$

(A1)

where $\varepsilon > 0$ is a small positive number, the $U$ may be used instead of $\delta g_{sc}$ for further calculations. Function $U$ can be written as:

$$U = \sum_{i=1}^{N} u_i,$$

where function $u_i$ determines the anomalous gravitational effect caused by the $i$-th elementary source. This conversion helps implement the sourcewise approximation of the original values of $\delta g_{sc}$.

We consider the case when the values of gravity terrain corrections $\delta g_{sc}$ are given at the nodes of a regular grid with the step $\Delta x = \Delta y = \Delta z$ over a rectangular domain of size $m \times n$. The correction values are stored in a rectangular matrix $G = [\delta g_{sc}^{ij}]: 1 \leq i \leq m; 1 \leq j \leq n$. The terrain corrections $G$ are approximated by the gravitational field $U$ of the net equivalent model, which consists of $n \times m$ spheres (mass points) located at each point $P_i$ ($P_i \in S$) with the coordinates $(x^i, y^i, z^i)$; the depth $b$ is selected lower than the surface $S$ ($\Delta \leq b \leq 2\Delta$), which ensures stability of the solution (Aronov 1976).
The anomalous gravitational effect of the approximation construction at the arbitrary point \( P \in S \) with the coordinates \((x, y, z)\) is given by:

\[
U(x, y, z) = \sum_{i=1}^{m} \sum_{j=1}^{n} \mu_{ij}^* u_{ij}^*,
\]

(A2)

where \( u^* = \gamma (z^* - z)/L^3 \) are harmonic functions that determine the gravitational field of the unit sphere \( V_z \) for \( \mu^* = 1 \); \( \gamma \) is the gravitational constant; \( L = \sqrt{(x^* - x)^2 + (y^* - y)^2 + (z^* - z)^2} \) is the distance between the centres of the spheres and calculation field point; and \( \mu^* \) are the source masses.

The masses of the spheres are determined by solving linear algebraic equations (SLAE) containing \( n \times m \) equations with \( n \times m \) unknowns

\[
UM = G,
\]

(A3)

where \( M = \{\mu_{ij}^*\} \) (1 \( \leq i \leq m \), 1 \( \leq j \leq n \), \( U = \{u_{ij}^*\} \) (1 \( \leq k \leq mn \), 1 \( \leq l \leq mn \)). The system (A3) can be solved by various methods that ensure the condition (A1), including the Seidel method, which has high convergence due to the strong diagonal dominance of the SLAE coefficients in the \( U \) matrix. The non-singular square matrix \( U \) is divided into two triangular matrices \( U = A + B \), allowing successive approximations \( M^k \) to the solution to be carried out according to the formula:

\[
AM^{k+1} + BM^k = G, \quad k = 0, 1, 2, \ldots,
\]

(A4)

where \( k \) is the iteration number.

When solving practical problems, SLAE (A3) usually has a large dimension (the number of unknowns \( mn \approx 10^4 \)) or an extra-large dimension (\( mn \approx 10^5 \)). A characteristic of the \( U \) matrix is a dramatic decrease in the values of the coefficients \( u_{ij}^* \) as the distance from the main diagonal \( i = j \) increases. For example, in the case of the plane surface \( S = S(x, y, z = 0) \) and the sphere with centres of height of 1.5 grid intervals \( \Delta \) (\( h = 1.5 \Delta \)), the following correlations are obtained: \( u_{ij}^*/u_{ij}^* \approx 0.0894; u_{ij}^*/u_{ik}^* \approx 0.00326; \) and \( u_{ij}^*/u_{ik}^* \approx 0.00021 \).

Obviously, it is possible to select a banded part \( \tilde{U} \) of the \( U \) matrix with a sufficiently narrow bandwidth \( 2r + 1 \) and consider \( u_{ij}^* = 0 \) during the iterative process (A4). The calculations are made according to formula (A2) for a certain cutoff value \( R_b \), depending on \( r \) (for \( R > R_b \), we consider \( u_{ij}^* = 0 \)). Depending on the choice of \( r \), the computational speed can increase by a factor ten or more. Thus recovery of the field \( U \) at the arbitrary point \( P \) reduces to solving the direct gravity problem prospecting from a given fragment of the approximation construction \( S^* \subseteq S \). The value of \( r \) can be chosen in a way that the contribution of sources not calculated and located in the domain \( S \setminus S^* \) will not exceed 1\%–2\% of the amplitude \( U \).

The algorithm described allows analytic approximation of gravity terrain corrections to be carried out quickly and with high accuracy by the iterative solution of SLAE of the form \( \tilde{U}M = G \) and to obtain the matrix \( M \) for the whole region of gravity research. A subsequent recovery of the values \( \delta g_{tc} \) at gravity stations of the grid is not difficult and is performed with an error not exceeding the value of \( \varepsilon \) in formula (A1).