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ГОСУДАРСТВЕННЫЙ
НАЦИОНАЛЬНЫЙ
ИССЛЕДОВАТЕЛЬСКИЙ
УНИВЕРСИТЕТ

Д. В. Шимановский

SYSTEM ANALYSIS

MICROECONOMIC MODELING AND SYSTEM ANALYSIS

Perm University Press

МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ
РОССИЙСКОЙ ФЕДЕРАЦИИ

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НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ»

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in the areas of Applied Mathematics and Computer Science
and Business Informatics

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Contents

INTRODUCTION

Currently, many citizens of the Russian Federation work in large companies incorporating thousands of structural units, each of which has its own goals and functions. It is a common situation if an employee of a particular structural unit knows only a small part of the company's business processes and is unable to evaluate their activities systematically. For example, a programmer is responsible for developing a software product to generate reports on the dynamics of mortgage loan sales for the management of PAO Sberbank. However, he/she has no idea what management decisions are made on the basis of these reports or what figures and why are included in this report.

In this regard, it is necessary that an employee perceives the organization he/she works in as a system and identifies his/her own role in its business activities, and also realizes how poor performance of his/her functions can affect the work of other structural units of the company.

These skills can be developed while studying a special academic subject called System Analysis. If we omit some formal ideas, it can be argued that system analysis is the science of systems. It studies the most general properties typical of any system.

Meanwhile, one can find systems not only in societies. Systems can also be found in both living and non-living things. Let us take a pack of wolves to illustrate the point. It has its own well-established structure, hierarchy, and each of its members performs their functions.

Thus, there is a need to develop training courses that could reveal the concept of a system in relation to a particular area.

In Perm State University there is a common practice of teaching relevant disciplines. Systems theory is taught as a separate course at the faculties of natural sciences (biology, information technology, etc.) and is not connected to any particular subject area. As for system analysis, it is taught at some faculties with reference to a certain scientific field (System analysis in IT, System analysis (for the students studying economics)).

This study guide is designed for students pursuing study tracks 01.03.02 Applied Mathematics and Computer Science and 38.03.05 Business Informatics of the Faculty of Economics of Perm State National Research University.

The purpose of studying the discipline is to develop system thinking skills as a tool of practical activity and to gain the ability of their practical application.

The object of this discipline is microeconomic systems.

The subject of the discipline is the processes of production, consumption and management in economic systems.

This study guide is designed for students who are supposed to have done the course of higher mathematics. The apparatus of differential calculus of functions of many variables, graphical analysis and the extremum theory are used by the guide as the most frequent methods of system analysis.

After completing the course a student will:

know the main aspects of the system methodology, which enables to investigate and solve complex poorly structured problems at different levels in different functional areas of organization management;

be able to put into practice modern methods of system analysis to perform specific tasks in the economic sphere;

have skills of structuring and formalizing analysis and synthesis problems during the study of complex economic systems.

The ***first chapter*** considers the fundamentals of system theory and of system analysis. In particular, it reveals basic mandatory and non-mandatory properties of systems, main categories of system analysis and the systems goal-setting theory.

The ***second*** and ***third*** chapters provide theoretical aspects of how system properties can be applied. This is illustrated by modeling market interaction. In particular, the theory of shaping market demand and market supply is revealed.

The ***fourth*** chapter includes theoretical material on modeling the main types of microeconomic systems, i.e. market structures, and considers aspects of modeling equilibrium in the market of perfect competition, and if there is monopoly or oligopoly.

The ***fifth*** chapter reviews the methods of mathematical modeling of microeconomic systems of a different type, namely, the labor market. In particular, the concepts of shaping of supply and demand in the labor market and ways to establish market equilibrium are considered.

The ***sixth*** chapter provides an overview of the general equilibrium theory in the inter-market interaction of certain economic systems.

CHAPTER 1. FUNDAMENTALS OF GENERAL SYSTEMS THEORY

1.1 The development of ideas about systems – historical background

The concept of system, which is the opposite of the concept of chaos, originated in ancient Greece. Based on rudimentary of knowledge of ancient Greek scientists, theologians of the Middle Ages considered man as a system of spiritual and material. The human soul (the spiritual origin) and the material body were inextricably linked. According to medieval philosophers, the God created the world intelligently, which means that the world is an ordered system.

Thus, ancient and medieval scholars believed that the world was an ordered and hierarchical system.

Later, scientists of New Age began to consider social systems as a separate type of material systems.

With time scientific research became more specialized and in the XVII century economics became a separeate discipline studying relations of production, consumption and distribution of economic goods. Market is one of the major economic systems as it is a mechanism of the interconnection between sellers and buyers.

At the end of the XIX century, due to the so-called marginalist revolution economics started to actively apply the apparatus of mathematical modeling. The concept of a limiting value was developed, which is mathematically reducible to the concept of a derivative.

Economists started to construct mathematical models of market relations and as a result, in the early XX century they began to pay special attention to the concept of equilibrium as a balance of two or more driving forces of market, which, thanks to self-regulation, come to a certain agreement.

Although rudimentary knowledge about the systemic nature of our world appeared as early as in ancient times, the theory of systems as a separate science was developed only in the 1950s when an Austrian biologist Ludwig von Bertalanffi, founded an institute for systems studies. L. von Bertalanffi focused on the study of general patterns of biological and social systems.

Subsequently, the achievements of the general theory of systems were applied in various fields of human activity: management, logistics, urban planning, public utilities.

Information technologies is the sphere where the general theory of systems has found special application. Currently, there are tens of thousands of computer networks in the world and there is one global network – the Internet. Computer networks work due to the performance of huge databases, which can be considered systems as well.

According to Rosstat, in 2019 the share of the Russian population using the Internet communication network was 82% of the total number of residents and the figure continues to grow rapidly.

The rapid development of social networks indicates that the application of the general theory of systems in the field of information technology is getting increasingly crucial. For example, the social network VKontakte is a complex system with a huge number of elements and connections between them.

Thus, although the foundations of systems thinking were laid in ancient times, it was only in the second half of the 20th century that the provisions of the general theory of systems became hot and happening.

1.2 The concept of the system and the main categories of general systems theory

The concept of a system dates back long before general theory of systems became an independent scientific discipline. It was used in biology, mathematics and some other sciences.

The concept of system is often associated with something ordered, interconnected, following certain rules (for example, the Mendeleev periodic system.

Translated from Greek, the word *σύστημα* means a whole consisting of several elements. Thus, while defining system, it is necessary to take into account that all its elements must have stable connections.

To date, authors have suggested many various definitions of a system. Here are a few of them:

- a set of interrelated elements;
- a set of elements which have a common goal;
- an ordered set of elements having a common structure.

The concept of a system is opposite to an incoherent set of elements that do not have a common goal or structure.

One of the central concepts of the general theory of systems is a system element. According to one definition, a **system element** is a single and indivisible part of a system. Indivisibility is considered as the inexpediency of further division of a system.

In terms of one science, an object is a system, while in terms of another science it is just an element of a more complex system. For example, in terms of geology the planet Earth is a complex system, while astronomy considers it as an element of the Solar System.

A system element can perform its own specific functions. This element is somehow different from the other elements of the system but at the same time is connected with them in one way or another.

Among all the elements of the system, one can determine systematically significant elements. **A systemically significant element** is an element whose features differ greatly from other elements of the system.

For example, if we consider federal subjects as elements of the Russian Federation system, its systemically significant elements are Moscow and St. Petersburg. They differ greatly from other regions by their GRP size, population density and some other socio-economic characteristics.

Systemically significant elements often have a special impact on the functioning of the entire system. For example, the largest companies in Russia influence the development of the entire economy of the Russian Federation.

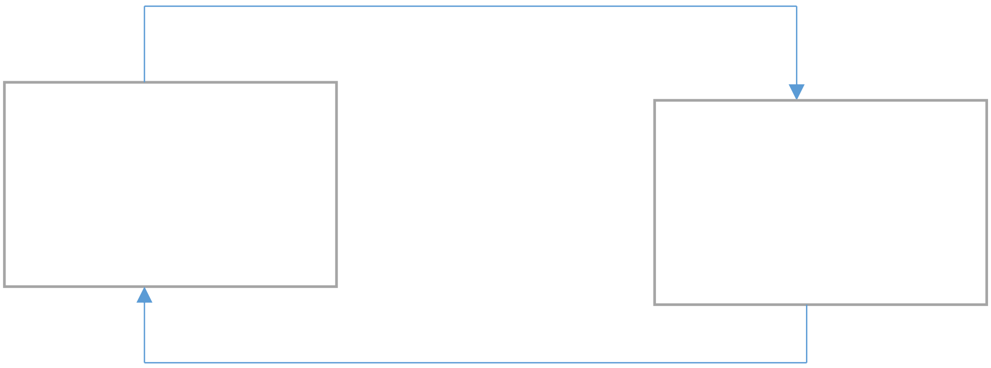
The second most important concept of the systems theory is system communication. **System communication** is the influence that a parameter of one element of the system has on the parameter of another element of the system. It should be emphasized that connection is not just a simple arrangement of elements one after another. Connection means that features of one element affect features of another element. For example, the more iron ore is supplied to a metallurgical plant, the more steel it will smelt.

System connections must be consistent and stable. If the connections between elements are random or episodic, such a set of elements cannot be considered a system.

For example, passengers in a bus are not a system because their connection with each other is temporary.

There are direct and reverse connections in a system (Fig. 1). The connection acting only in one direction is direct. However, it is reverse connections that are very important to study.

a) Example of a direct connection



b) Example of a reverse connection

Figure 1.1. Types of connections in a system

Due to reverse connections, system elements adapt to each other's work. For example, according to the classical methodology of economic theory, three elements can be identified in the national economic system: government bodies, population and business community. Government bodies conduct public surveys to find out how much the population is satisfied with the economic situation in the country. By receiving feedback from the population, the government can adjust its economic policy.

For example, this is how the decision on a change in the key interest rate by the Bank of Russia was taken. If aggregate demand increases, there is a frequent increase in inflation in the country. In this case, the Central Bank takes a decision to raise the key rate. However, an economic response to this management decision is not immediate. It takes two or three quarteres for the Bank of Russia to receive a feedback from the real sector of the economy. This feedback comes in the form of figures for GDP growth or inflation.

The third most important category of the systems theory is the concept of a subsystem. **A subsystem** is a divisible part of a system. A subsystem can consist of elements.

Below is the description of system classifications according to the main classification features.

1.3 Basic classifications of systems

Systems are classified according to a particular feature. Currently, there is no single generally accepted classification of systems. Below are the most well-known and important system classifications from the point of view of the author of this teaching guide.

According to the degree of openness, the systems are divided into ***open*** and ***closed***. Open systems continuously interact with external environment. For example, any commercial enterprise is an open system. New employees are employed by the company from external environment. Besides, many commercial enterprises interact with partners, suppliers and customers who also belong to external environment. That is why, it is necessary to give a clear definition of external environment. ***External environment*** is a set of objects that are not parts of the system but have an impact on it. For example, customers of a bank are not its elements, but they bring it the main income.

According to the degree of organization, there are ***poorly organized***, ***well-organized*** and ***self-organizing*** systems. Unlike poorly organized systems, in well-organized systems, many processes are repeated according to a given algorithm.

Special attention is paid to self-organizing systems. Many biological systems are self-organizing ones. This is because in biological world many species evolve and adapt to changing environmental conditions.

Self-organization is generally inherent in social systems as humanity is gradually developing economically and socially.

According to the degree of complexity, there are ***simple*** and ***complex*** systems. The more elements and connections a system has, the more complex such a system is. However, it is usually impossible to determine the point at which a simple system becomes a complex one.

By their origin, systems can be ***natural*** and ***artificial***. Artificial systems, as follows from the name, are man-made. Examples of artificial systems are a computer network or a car.

There are also ***static*** and ***dynamic*** systems. If features of a system are constantly changing, such system is called dynamic. For example, any company is a dynamic system, since its financial indicators (profit, revenue, return on assets) are constantly changing.

However, the above information allows a conclusion that many ways of classifying systems are conditional. For example, there are no pure self-organizing systems, since any system develops both under the influence of its own active forces and as a result of interaction with the external environment. Similarly, there are no pure closed systems.

1.4 Main properties of systems

Each system has its own unique properties. For example, the property of Perm city as a system is its location on two banks of the Kama River. Nevertheless, scientists identify a number of mandatory properties that each system must possess. Besides, there are properties that are common to most systems.

The first mandatory property of a system is **integrity**. According to the integrity property, each of the elements of the system is connected with at least one of its other elements.

Here is a simple example. Suppose that a particular ant was a part of the anthill system. However, at some point it did not return to its anthill. Consequently, it lost connection with all the other elements of the system. Thus, according to the integrity property, it is no longer part of the anthill system.

Another mandatory property of a system is **synergy**. The idea behind synergy is that the efficiency of the entire system is higher than the sum of the efficiencies of its individual elements. In other words, an element inside the system works more efficiently than it would work outside it.

Here is an example. Let's take two independent freelance programmers. The average monthly income of the first one was 40 thousand rubles, and the average income of the second one was 50 thousand rubles, but when they started working as a team, their total income reached 130 thousand rubles.

The question arises: why did the total income become 130 thousand rubles but not 90 (40 + 50) thousand rubles? Possible resons are the exchange of experience, elimination of duplicate functions, division of labor, etc. Thus, a **synergistic effect** occurs which in this case equal to 40 thousand rubles.

If there is no synergistic effect when arranging elements into a system, it makes no sense to organize them into a system.

The third mandatory property of a system is **emergence** (from English *emergent* – arising, unexpectedly appearing). According to the emergence property, a system possesses the properties which its individual elements do not have.

For example, a car is a complex system with many elements. But none of its elements individually is capable of transporting cargo or passengers. Only when the elements are combined into one whole, the resulting system can perform its main function.

Similarly, no element of an automobile plant alone can produce finished cars.

In addition to the mandatory properties of systems, there are also optional properties, possessed by not all, but by most systems.

The first of these properties is the **robustness** property (English *robustness*, from *robust* – strong, strong, firm, stable). It means that if individual elements fail, the whole system continues to function. For example, if an organization leaves a group of companies, generally the group continues to function.

The second optional property is the **equifinality** property (from English *equifinality*). The equifinality property means that a system is able to reach its final state regardless of its initial state or environmental conditions.

In terms of philosophy, all systems have this property because sooner or later any system will cease to exist regardless of its initial state and environmental conditions.

Thus, we have considered the main system properties. Now let's consider such an important category as the system goal.

1.5 The Theory of Goal-setting of Social Systems

The development of social systems is often subordinated to some goal. The actions of a person should have a goal too. It is believed that if a person spends most of their time aimlessly, this may indicate that they have serious psychological problems.

Education should also be a goal-directed process. When applying for a chosen direction of study in higher education, the applicant must clearly understand where this knowledge can be useful and what professional positions they can apply for after graduation.

Often the choice of unattainable goals leads to a decrease in the effectiveness of the system (for example, the construction of a world system of socialism as the Soviet Union's goal).

Currently, a methodology for setting goals called SMART (Specific, Measurable, Attainable, Relevant, Time-bound) has become widespread.

According to this methodology, a goal must be:

1) specific;
2) attainable;
3) measurable;
4) relevant;
5) time-bound.

Next, let's take a closer look at each of the goal's extensible properties.

Specificity. The goal should be specific, so that at the end of the target period it can be estimated whether the system has achieved the goal or not. For example, such goals as "I want to be rich", "our company should become recognizable throughout Russia", "we want to make our customers happy" are not specific.

Attainability. When setting a goal, an individual must think about what means and ways it can be achieved by and whether it is likely to be achieved.

Measurability. The statement of a goal must include clear quantitative indicators.

Relevance. The goal should not contradict a person's life values and priorities.

Time-binding. When a person sets a goal, they must be clearly aware of the time by which it will have been achieved.

Here is an example of a poorly formulated goal: "I want to be rich". A person has a desire to become rich, to meet certain needs and, possibly, to reach a certain social status. However, this goal satisfies none of the above criteria.

Let's consider a more successful statement of the goal in terms of SMART methodology: "I want to have a deposit account with a balance of 1 million rubles in PAO Sberbank by January 1, 2027. To achieve this, I need to get a job no later than January 1, 2022 with a salary 40 thousand rubles minimum, of which I have to deposit at least 20 thousand rubles per month ".

This goal is more specific, includes certain figures and has means of achievement.

Depending on the scale and forecasting horizon, there are **tactical** and **strategic** goals. An example of a tactical goal for a Russian person with an average income can be a purchase of some household appliance. An example of a strategic goal for the same person is buying an apartment.

The achievement of tactical goals should not interfere with the achievement of strategic goals. Ideally, tactical goals should specify strategic ones.

The goal should be ambitious, but at the same time achievable. The situation when the goal is achieved by itself is unacceptable. For example, a situation when a healthy twenty-year-old boy sets a goal of running a hundred meters in any time period is unacceptable.

A goal also should solve a problem. For example, it is foolish to save money for nothing like it was given in the example above. Sooner or later, money must be spent on satisfying some need.

These are the main features of the general theory of systems. Later, chapters 2-5 will explore the system of economic and mathematical disciplines in terms of their systemic interrelation.

SELF-CHECK QUESTIONS FOR CHAPTER 1

1. Explain how a system differs from a group of random elements.
2. Explain the emergence property.
3. Give an example of a reverse connection in a system.
4. What is the difference between open and closed systems?
5. What properties should a goal have according to SMART methodology?
6. Give an example of one of your goals which can be achieved within no more than a month.
7. Give an example of synergistic effects that occur in the systems you know.

SELF-CHECK TEST FOR CHAPTER 1

1. What are the two most typical features distinguishing a system from a group of random objects:
    a) the system has stable connections between the elements;
    b) the system can accept new elements;
    c) all elements of the system have a common goal;
    d) the system does not always have a material representation.
2. What is the difference between an open and closed system?
    a) an open system continuously interacts with external environment;
    b) an open system can accept new elements;
    c) an open system is connected with other systems;
    d) an open system has no clear boundaries between its elements and external environment.
3. What is the difference between a static and dynamic system?
    a) a dynamic system changes qualitatively over time;
    b) a dynamic system is capable of moving;
    c) the features of a dynamic system are constantly changing;
    d) a dynamic system is capable of independent development.
4. Can a subsystem be divided into elements?
    a) yes, because a subsystem is a divisible part of the system;
    b) no, because a subsystem is an indivisible part of the system;
    c) yes, because any system has a hierarchy of "system – subsystem – element";
    d) no, because a system can consist of subsystems, but it cannot consist of elements.
5. The emergence property implies that:
    a) the whole system has properties that individual elements do not have;
    b) each element of a system has its own unique properties;
    c) a system changes its characteristics over time;
    d) a system always comes to the same final state.
6. The robustness property means:
    a) the system keeps working even if some of its elements fail;
    b) the system ceases its existence if some of its elements fail;
    c) the relationship of all elements of the system with each other;
    d) the system has clear boundaries.
7. The goal-setting law states:
    a) the goal of the system development is determined by objective natural and social laws;

b) each system must have a major goal;
c) the system must have a hierarchy of goals;
d) the goal of the system should be clearly stated.

8. According to the feedback principle, the original element:
    a) changes other elements of the system;
    b) changes itself;
    c) changes objects from the external environment;
    d) changes the nature of the connections between elements.
9. Which of the methods of system management implies that the system has only qualitative development goal?
    a) reactive method,
    b) target management method,
    c) method of stimulating effects,
    d) stereotypical management method.
10. The system is supposed to be in critical condition if:
    a) it is unable to perform its functions in full,
    b) it lost a significant part of its elements,
    c) connections in the system have become unstable,
    d) the system will soon cease to exist.

CHAPTER 2. MATHEMATICAL MODELING OF CONSUMER BEHAVIOR

In the early stages of the development of economics as a science, it hardly ever used mathematical apparatus. The marginalist revolution of the end of the XIX century made an important contribution to wide applicability of mathematical methods in economic theory. Starting from that period, the explanation of economic processes took place using the language of mathematics. Limiting values, the calculation of which is reduced to the calculation of partial derivatives, became of special theoretical value.

The first paragraph of this chapter considers models from the theory of consumer behavior, the second paragraph looks into those of manufacturer behavior. The third paragraph contains an overview of economic and mathematical models describing the types of market structures.

2.1 Modeling consumer behavior in the market

From the course of economic theory it is known that one of the main assumptions of neoclassical microeconomics is that a consumer behaves rationally. In other words, they prefer the best commodity set available.

This assumption provides the basis for the use of optimization models in microeconomic theory. Meanwhile, theoretical economists recognize that this assumption is not always realized. Nevertheless, the acceptance of this most important assumption allows the use of mathematical tools.

In addition to this assumption, according to the microeconomic theory, there are some other assumptions of consumer behavior in the market.

The first of them is ***transitivity axiom.*** It states: if for some consumer commodity set A is more preferable than commodity set B, and commodity set C is more preferable than set B, then, consequently, for the consumer, commodity set C is more preferable than set A.

Further, preference ratio will be represented by symbol "$\prec$". Thus, expression "A $\prec$B" will mean that commodity set B is preferred by the consumer to commodity set A.

The second assumption is ***complete orderliness axiom***. It means that either commodity set *A* is more preferable than commodity set *B*, or commodity set B is more preferable than commodity set *A.* In a formal language , this expression can be written as follows: either $A \prec B$, or $B \prec A$.

The idea behind this axiom is that a consumer is completely confident in their preferences. They can always determine what is best for them without too much speculation.

The third assumption is called ***reflexivity axiom***. It means that $A \prec A$.

This teaching guide mentions only the basic axioms of consumer choice. They are the basis of the theory of consumption.

Now let's consider the definition of the concept of utility, which is the basis of the theory of consumption. According to one of the definitions, ***utility*** is the property of a good to satisfy a particular need.

According to marketing theory, need is the basis of any product. Thus, each product satisfies some need of an individual. For example, a clock satisfies the need for information about the exact time. A car satisfies the need to get around.

The more needs a particular product satisfies, the higher its utility. Generally, letter *U* stands for utility (from English *utility*).

In terms of modern concepts of microeconomics, utility is a quantity that cannot be measured. Those who know laws of physics of physics well might find it weird. However, not all modeling methods used in physics are applicable in economics.

Another important feature of consumer choice theory is the concept of ***utility function***. If *X* is a number of commodity sets and *R* a the set of real numbers, the utility function is $u(X)$ a representation of the type $u: X \to R$ for which the following property is met: if $X \prec Y$, then $u(X) < u(Y)$.

If a commodity set consists of more than one product or service, the utility function is a function of many variables.

The third most important definition of consumer choice theory is the concept of ***marginal utility***. Marginal utility is represented by MU_X. According to the definition, ***marginal utility of product X*** is an indicator showing an increase in total utility if the consumption of product ***X*** increases by one. If we assume that commodity X is infinitely divisible, then the definition of marginal utility reduces to a partial derivative:

$$MU_X = \frac{\partial U}{\partial X}, \qquad (2.1)$$

where MU_X is the marginal utility of the product *X*, U is the overall utility level, *X* is the quantity of product *X*.

The concept of marginal utility is closely connected with the law of decreasing marginal utility, according to which, when the consumption of product *X* increases, its marginal utility tends to decrease. A graphic illustration of this law is shown in Figure 2.1.

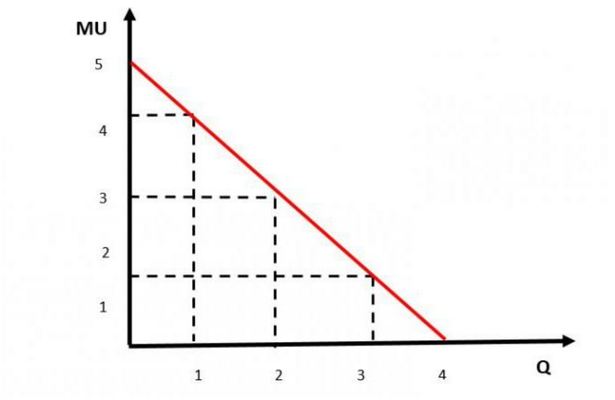


Fig.2.1. Graph of decreasing marginal utility

Here is an example of how this law can be applied to everyday situations. Let *X* be the number of meals in a restaurant per month. If a person has a relatively modest income, one meal in a restaurant a month will bring them a lot of positive emotions. If a person can afford to have a meal in a restaurant twice a month, the effect from the second visit to a restaurant will not be as strong as from the first one. The trend will continue until going to restaurants for the consumer becomes commonplace and then it stops being beneficial.

But how can this be? Can a product become anti-beneficial? It really can. Such products are called ***anti-goods***. A logical question arises: if anti-goods do not benefit the consumer, why do they exist? The answer is not too obvious. For example, alcohol, drugs and cigarettes are obvious anti-goods. However, favourable and unfavourable effects of these goods are spread over time. An adolescent smoker is unlikely to consider the negative consequences of smoking for their health which they may get after a few decades.

Let's assume that commodity set *A* only includes two products – *X* and *Y*. With different combinations of products *X* and *Y* the consumer can achieve similar utility rate. Let's depict this graphically (Fig. 2.2).

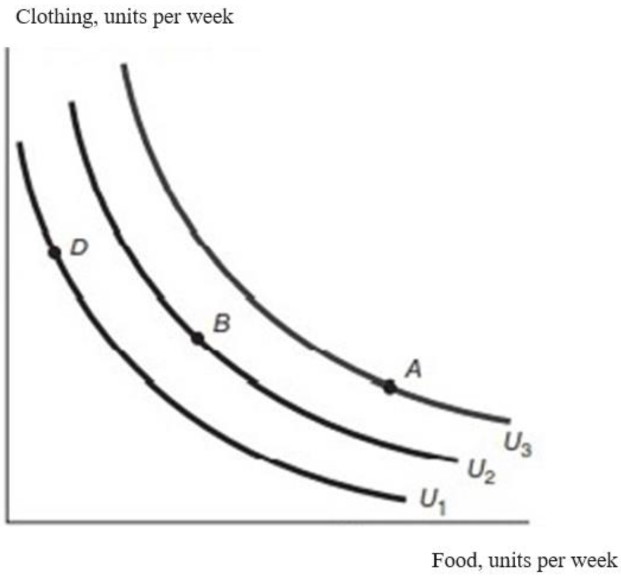


Fig.2.2. The map of indifference curves

Let's assume that product *Y* is clothing. The value of this product is plotted along the abscissa in the Cartesian coordinate system. Product *X* is food. Its value is plotted along the ordinate axis. The fact that these products are actually not goods, but groups of goods should not be confusing. Let's consider a master's student who lives with his parents and spends money only on food and clothes. He can buy better food, but cheaper clothes. Another option is that he can wear clothes of more expensive brands, but buy food of lower quality. In both cases, the utility levels will be the same.

Thus, we come to the definition of an indifference curve. ***An indifference curve*** is the geometric locus of points in *X*–*Y* coordinates showing different combinations of two goods providing equal utility to the consumer.

Look at Figure 2.2. It shows three indifference curves. They show the same utility levels provided by certain combinations of food and clothing consumption. The higher the indifference curve is located, the higher the utility level it shows. For example, Figure 2.2 shows the indifference curve U_3 representing a higher utility level than a similar curve U_2.

Another important concept of the theory of consumption is marginal rate of substitution. ***Marginal rate of substitution of product X with product Y*** (marked $MRS_{X,Y}$) is a value that shows the rate at which it is necessary to increase the consumption of good X while reducing the consumption of goods Y.

Marginal rate of substitution is calculated by formula:

$$MRS_{X,Y} = \frac{MU_X}{MU_Y}, \qquad (2.2)$$

where $MRS_{X,Y}$ is marginal rate of substitution of good *Y* by good *X;* MU_X is marginal utility for good *X*; MU_Y is marginal utility for good *Y.*

Example 2.1. The preferences of a senior student Sergey Petrov can be described by utility function $U(X,Y) = X^2Y$, where *X* is the number of visits to the swimming pool per month, *Y* is the number of visits to the cinema per month. Find:

а) marginal utility values for goods *X* and *Y*;
б) marginal utilities for goods *X* and *Y* at point (3;5);
в) maximum rate of substitution of visiting a swimming pool with visiting the cinema.

Solution

а) we calculate marginal utilities using formula (2.1):

$$MU_X = \frac{\partial U}{\partial X} = 2XY,$$

$$MU_Y = \frac{\partial U}{\partial Y} = X^2,$$

б) substitute this point into the resulting values:

$$MU_X(3;5) = 2*3*5 = 30,$$

$$MU_Y(3;5) = 3^2 = 9;$$

в) we calculate marginal rate of substitution using formula (2.2):

$$MRS_{X,Y} = \frac{2XY}{X^2} = \frac{2Y}{X}.$$

2.2 Types of utility functions

Each individual has their own utility function for various goods. However, for theoretical purposes, it is useful to determine the most common utility functions.

The ***Cobb–Douglas utility function*** is the most popular function used in scientific and applied research. It describes a common situation: two goods can replace each other, but the marginal rate of substitution is different at different points. Suppose that a person has a choice: to spend money on food or on clothes (Fig. 2.2). If a person spends a lot of money on food and not enough money on clothes, his need for food appears to be sufficiently satisfied. At the same time, his need for clothes is not satisfied enough. At this point, the marginal rate of substitution of food with clothing (*MRS*) will be high. If the situation is opposite: a person spends much money on clothes and a little on food, *MRS* will be lower (Fig. 2.3).

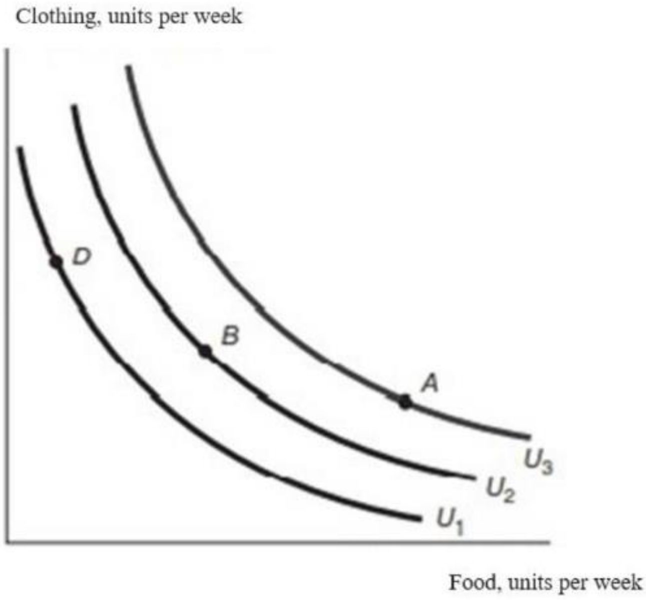


Fig.2.3. The map of indifference curves for the Cobb – Douglas function

Analytically, utility function can be as follows

$$U = X^{\alpha} Y^{\beta}, \tag{2.3}$$

where U is the total utility rate; X is the quantity of the first good in the commodity set; Y is the quantity of the second good in the commodity set.

The second type of utility functions is the ***utility function for goods which are perfect substitutes***. In economic theory, perfect substitutes are goods that can substitute each other in equal proportion. For example, a student does not care which color pen to use, black or blue.

The graph of indifference curves for this utility function is shown in Figure 2.4.

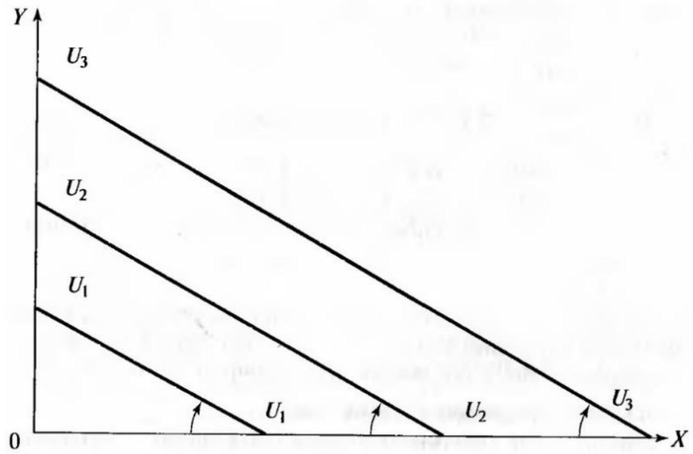


Figure 2.4. The map of indifference curves for goods which are perfect substitutes

As follows from Fig. 2.4, the indifference curves for the given type of goods are represented by straight lines. *MRS* for this case is the same at each point for a particular indifference curve.

Analytically, the utility function for this case will be as follows:

$$U = aX + bY. \tag{2.4}$$

The third common type is the utility function for ***complementary goods***. Economic theory defines complementary goods as those which complement each other. Hence, goods that lack consumer properties without each other are called perfect complements.

The map of indifference curves for perfect complement goods is shown in Figure 2.5.

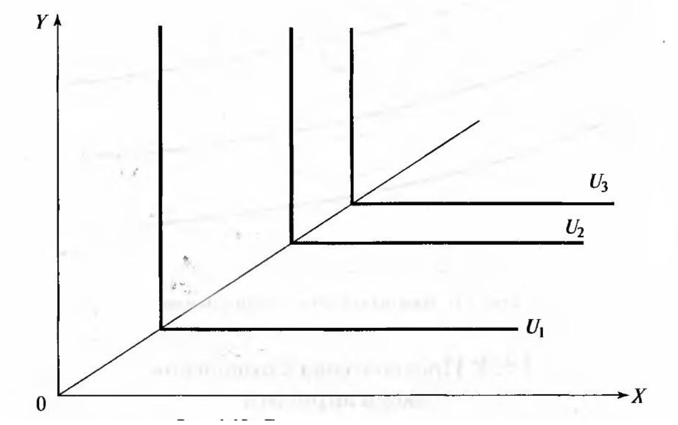


Figure 2.5. The map of indifference curves for goods which are perfect substitutes

As can be seen from Figure 2.5, indifference curves are represented by broken lines. Here is an example explaining this. Suppose that a small company has five system units and five computer monitors in its office. They provide some utility to the company. Now let's assume that the situation has changed with the company getting another system unit. Now it has six system units and five monitors. Despite this, the utility the company receives has not changed.

Analytically, the utility function for this case will be as follows:

$$U = \min\{aX, bY\}. \tag{2.5}$$

This formula confirms the above. If, among a pair of complementary goods, the quantity of one exceeds the quantity of the other, the utility received by the consumer will be the same as with the equal quantity of both goods.

The last type of utility function considered in this manual is the utility function for goods and anti-goods. The concept of anti-goods was defined above. Frequently a person has to consume some kind of anti-good as a sort of "compensation" for some extra utility and satisfying the needs.

Example 2.1. Twenty-five-year-old Yuri Petrov does not like to come to work every day at 9:00. To do so he has to go to bed no later than midnight. In order to do so, it is advisable not to eat after 22:00 and not to drink coffee after 18:00. But Yuri gets so tired at work. He often wants to watch TV or scroll through the feed on social networks, postponing the preparation of dinner for later. But work brings him an income. Thanks to this money, Yuri Petrov can rent a studio apartment, buy food, clothes and spend some money on entertainment.

In example 2.2, work for Yuri Petrov is an anti-good, and the consumption of goods and services is a good.

An approximate view of the indifference curve map for the case of a good and that of anti-good is shown in Fig. 2.6.

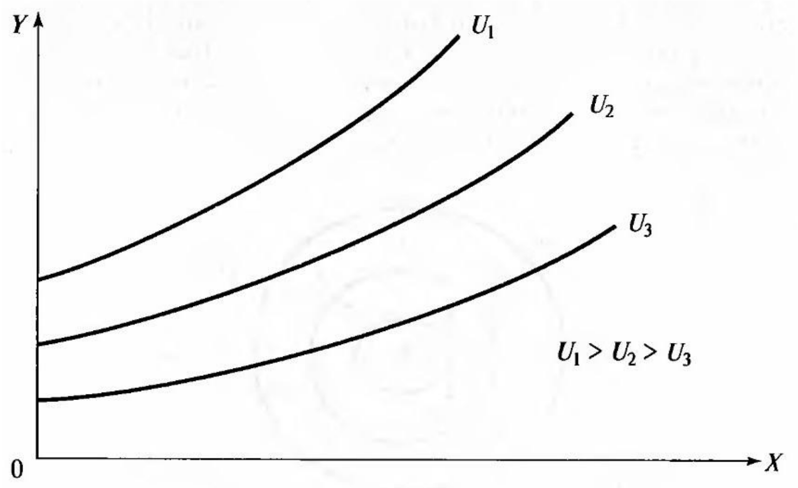


Fig.2.6. The map of indifference curves for cases of a good and anti-good.

In the graph shown in Figure 2.6, *Y* is a good, and *X* is an anti-good. If we move from the point where Y = 0 along the 0Y axis, the utility level will be increasing. Hence, *Y* is a good.

2.3 Budget constraint line and consumer choice

The income of almost any person is limited. Distributing their income between various goods and services, the consumer makes a choice in favor of this or that product group.

The budget constraint model does not reveal the theory of consumer choice. It only provides a visual interpretation of changes in an individual consumer's budget possibilities depending on the current situation.

Suppose that a consumer spends his money on only two groups of goods (for example, clothing and food). Let's them be groups *X* and *Y*, *respectively* . Let the price of product *X* be P_X , and the price of product *Y* be P_Y .

The consumer's income will be represented by *M*. Then the budget constraint equation takes the form

$$P_X X + P_Y Y = M. \quad (2.6)$$

Graphically, the budget constraint equation can be represented as a straight line, shown in Figure 2.7.

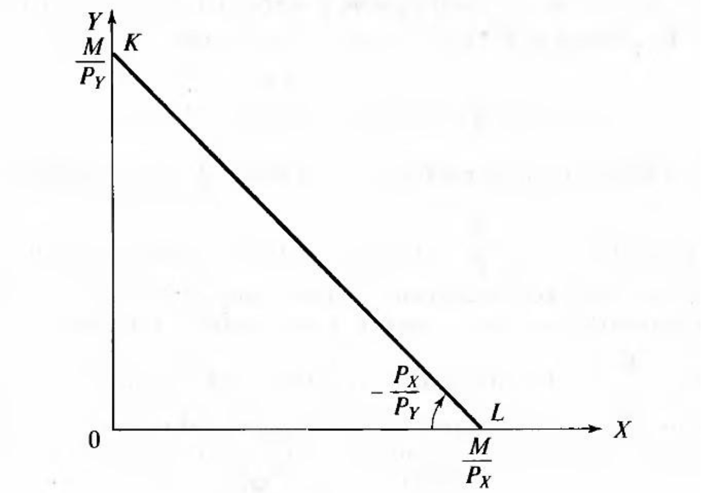


Fig.2.7. Graphical representation of the budget constraint line

Ratio $\frac{P_X}{P_Y}$ characterizes the slope of the budget constraint line. This straight line intersects axis 0 Y at point $\frac{M}{P_Y}$, and axis 0 X at point $\frac{M}{P_X}$.

Now let's assume that the consumer's income has increased. What in this case will happen to his budget constraint line? It will shift right-side-up. A graphical interpretation of this situation is shown in Figure 2.8.

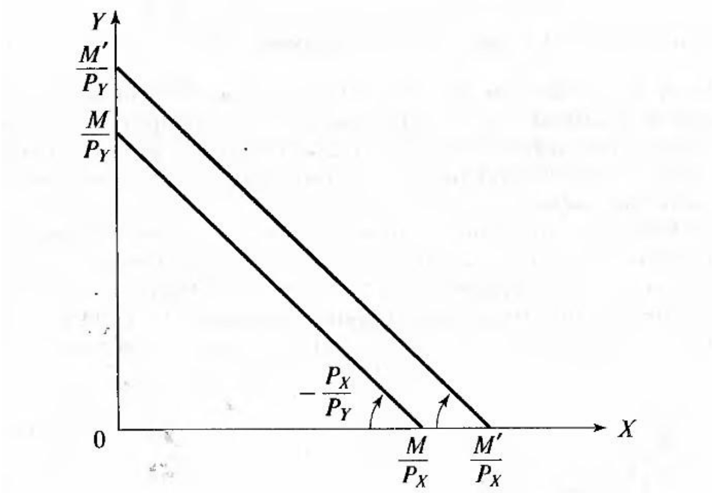


Fig.2.8. Graphic representation of the budget constraint line shift with an increase in a consumer income

Figure 2.8 shows that the indifference curve slope does not change with an increase in income.

Any point that lies inside the triangle formed by the budget constraint line and the coordinate axes is accessible for the consumer. If the point is outside this triangle, it means that the consumer's income is insufficient to purchase the given commodity set.

Neoclassical microeconomic theory believes that a consumer seeks to maximize the utility that he receives from the consumption of goods and services. However, the consumer's financial resources are limited. Therefore, he distributes his income in such a way that, in conditions of limited resources, he gets the greatest utility.

Mathematically, this is a nonlinear programming problem:

$$U(X,Y) \to \max, \tag{2.7}$$

$$P_X X + P_Y Y = M. \tag{2.8}$$

Problem (2.7)–(2.8) belongs to the class of nonlinear programming problems due to the fact that in most cases the function $U(X,Y)$ is nonlinear.

Graphically, the consumer's optimum is the point where the budget constraint line touches the indifference curve (Figure 2.9).

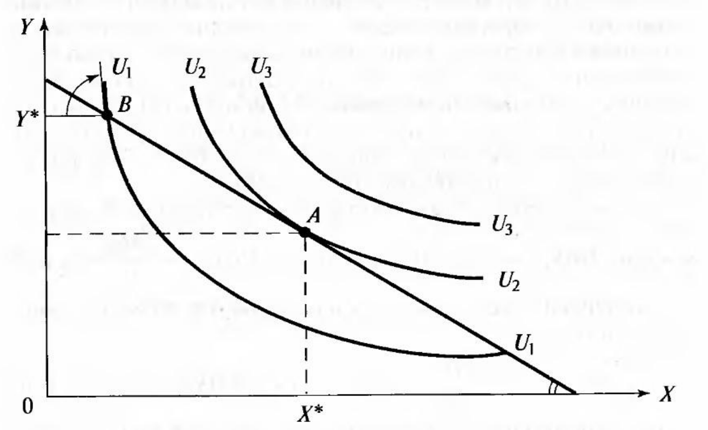


Fig.2.9. Graphical interpretation of the consumer's optimum

The graph in Fig.2.9 shows that utility level U_3 is not available for the consumer due to his lack of cash. Meanwhile, the consumer can achieve a higher utility level than the one represented by the indifference curve U_1. Point A on graph 2.9 shows the ***optimum of the consumer***. At the consumer's optimum point , the following ratio is satisfied:

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}. \tag{2.9}$$

Example 2.3. A fourth-year student Yuri spends money on lunches in the university canteen and on classes with a personal trainer in the gym. Student Yuri's preferences can be represented by the following utility function: $U = X^{0,7}Y^{0,2}$, where X is the number of classes with a personal trainer, Y is the number of lunches in the university canteen. One lesson with a personal trainer costs 700 rubles. One lunch in the canteen is 200 rubles. Yuri's monthly income is 9000 rubles. Find the optimal number of lunches in the university canteen and individual classes with a personal trainer for student Yuri.

Solution. According to the formula (2.9), the consumer's optimum is reached at the point where equality $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$ is satisfied. In our case $P_X = 700 P_Y = 200\ M = 9000$. Thus, student Yuri's budget constraint line will have the form

$$700X + 200Y = 9000\ .$$

Next, we calculate the marginal utilities:

$$MU_X = \frac{\partial U}{\partial X} = \frac{0,7Y^{0,2}}{X^{0,3}}.$$

$$MU_Y = \frac{\partial U}{\partial X} = \frac{0,2X^{0,7}}{Y^{0,8}}.$$

Then

$$\frac{MU_X}{MU_Y} = \frac{0,7Y^{0,2}Y^{0,8}}{0,2X^{0,3}X^{0,7}} \frac{7Y}{2X} = \frac{700}{200} = \frac{7}{2}.$$

Next

$$14X = 14Y,$$

$$X = Y.$$

Substitute the resulting value into the budget constraint line:

$$700X + 200X = 900,$$

$$X = 9000,$$

$$X = 10, \qquad .$$

$$900Y = 9000,$$

$$Y = 10.$$

Answer: for student Yuri, 10 lunches in the student canteen per month and 10 classes with a personal trainer will be optimal.

2.4 Demand function and its properties

Consumer preferences shape their demand for goods and services. The consumer's optimum point characterizes an individual consumer's demand for goods *X* and *Y* but not their actual consumption.

Let's provide a more formal definition of the concept of demand. **Demand** is the desire and ability of a consumer to buy a certain amount of goods at a certain price. Demand means not only the desire to purchase something, but also the ability to do it. For example, an unemployed person who has no substantial savings cannot create demand for luxury real properties.

Here is a graphical interpretation of the dependence of the demand function on consumer preferences, which is expressed by the utility function (Fig. 2.10).

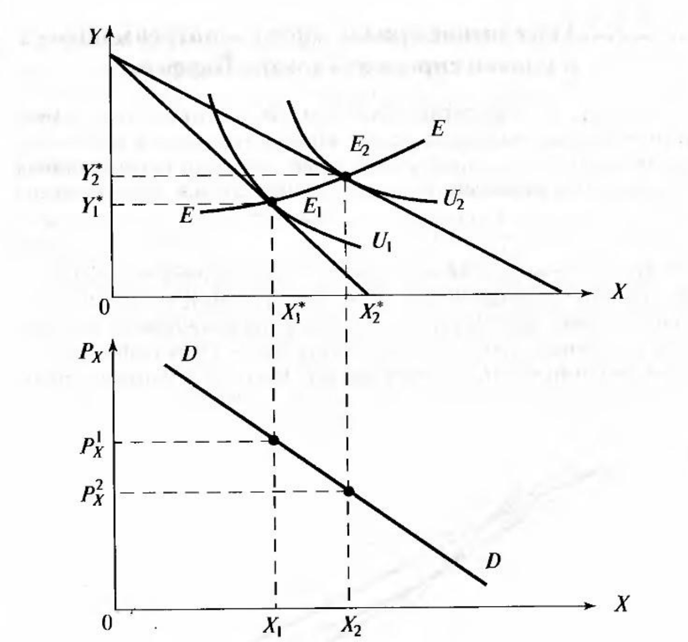


Fig. 2.10. Consumer preferences and demand function

Let's consider Figure 2.10 in detail. The upper graph shows two budget constraint lines. The first line represents budget restriction before product X price reduction, while the second line represents the same but after the reduction of the price. Thus, points $E_1$and E_2indicate relevant optima of the consumer. They, in turn, form the demand for goods *X* and *Y*.

The lower graph in Fig.2.10 shows two points of demand for product *X*: before and after the reduction of the price for this product. The straight line drawn through these two points indicates the demand for product *X*.

Thus, the demand function of a particular consumer for a product depends on the price of this product, the prices of substitute goods and the income of this consumer. Analytically form, this can be represented by the following formula

$$D_{Xi} = D_{Xi}(P_X, P_Y, M_i), \tag{2.10}$$

where $D_{Xi}$is the demand of the *i*-th consumer for the product *X;*M_i *is* the income of the *i*-th consumer.

The function of individual demand is homogeneous to zero degree. In the mathematical language this can be described as follows:

$$D_{Xi}(tP_X, tP_Y, tM_i) = D_{Xi}(P_X, P_Y, M_i) \quad \forall t. \tag{2.11}$$

In other words, demand is independent of the monetary units in which prices and consumer income are represented. For example, when there is a change of currency in the country or after a denomination, the consumer demand for goods and services does not change.

Example 2.4. The utility function of a consumer has a form $U = X^{2/3}Y^{1/3}$. Derive the demand functions for goods *X* and *Y* for these preferences.

Solution. Since the demand is determined by the consumer optimum, let's use the formula (2.9), where prices are unknown parameters.

$$MU_X = \frac{2Y^{1/3}}{3X^{1/3}},$$

$$MU_Y = \frac{1X^{2/3}}{3Y^{2/3}},$$

$$\frac{2Y^{1/3}3Y^{2/3}}{3X^{1/3}X^{2/3}} = \frac{2Y}{X} = \frac{P_X}{P_Y},$$

$$2YP_Y = XP_X,$$

$$3YP_X = M,$$

$$Y = \frac{M}{3P_X},$$

$$\frac{3}{2}XP_X = M,$$

$$X = \frac{2M}{3P_X}.$$

Answer: for these consumer preferences, the demand function for product *X* is $X = \frac{2M}{3P_X}$, and for product *Y* it is $Y = \frac{M}{3P_X}$.

Let's analyze in detail solution for example 2.4. As you can see, the demand functions for goods *X* and *Y* in X–$P_X$coordinates are hyperbolic. This is quite natural: if an individual's preferences can be described by the Cobb–Douglas utility function, the demand functions are hyperbolic.

Next, as is seen, the volume of demand positively depends on the individual's income and negatively on the price of this product. It can also be noted that in the case of Cobb–Douglas preferences, the demand for one of the goods does not depend on the price of the other product. Hence, for Cobb–Douglas preferences, goods *X* and *Y* are not substitutes.

In Example 2.4, the volume of demand positively depends on the consumer income. This is typical for most products. However, some economic goods do not obey this law.

According to the reaction of demand for goods when the consumer's income increases, economists distinguish three types of economic goods.

If, with an increase in consumer income, the demand for a product increases, such a product is considered ***inferior***, or ***low-quality***. Examples of such goods are cheap products (bread, sausage, potatoes). Cereals and potatoes were the basis of the diet of Russian peasants in early 20th century. However, with the general growth of incomes, the diet of an average Russian person became more diverse. Demand for potatoes and cereals fell.

Analytically, the condition under which a product can be considered low-quality is as follows:

$$\frac{\partial D_X}{\partial M} < 0. \qquad (2.12)$$

If, with an increase in income, the demand for a product grows slower than the income, such a product is called normal. Despite the fact that with an increase in income, the demand for these goods increases, its share in consumer spending decreases.

Example of normal goods are clothing and shoes. Up to a certain time, spendings on clothes and shoes increased along with the increase in incomes. Clothes became more diverse and fashionable. But since the late 2000s, many Russian companies have relaxed the requirements for a corporate dress code. Spending on clothing began to fall, despite growing incomes.

Analytically, the condition under which a product is considered normal is expressed by the formula

$$0 < \frac{\partial D_X}{\partial M} < 1. \qquad (2.13)$$

If, with an increase in income, the demand for a product grows faster than the income, such a product is called ***luxury good***. The share of spending on luxury goods in their overall structure increases with the growth of consumer income. Luxury goods include cars and residential real estate. Generally, luxury goods are at the top of the pyramid of human needs.

Analytical form of the condition under which a product is considered a luxury good:

$$\frac{\partial D_X}{\partial M} > 1. \qquad (2.13)$$

Example 2.5. The dependence of the demand for product *X* on the amount of consumer income is found: $D_X = 3M^2 - 2M - 5$. Under what conditions can the product be considered normal?

Solution. According to the formula (2.13), the product is normal if the condition$0 < \frac{\partial D_X}{\partial M} < 1$ is met. Let's find the derivative of the demand for product *X* on income: $\frac{\partial D_X}{\partial M} = 6M - 2$.

Next, we find the income area *M* satisfying two inequalities. Let's start with inequality $\frac{\partial D_X}{\partial M} > 0$.

$6M - 2 > 0,$

$M > \frac{1}{3}$.

Let's consider the second inequality $\frac{\partial D_X}{\partial M} < 1$

$6\mathrm{M} - 2 < 1,$

$$6M < 3,$$
$$M < \frac{1}{2}.$$

Answer: the product is normal with the amount of income from a $\frac{1}{3}$monetary unit to a $\frac{1}{2}$monetary unit.

2.5 Income effect and substitution effect

As is known, the demand for most economic goods negatively depends on their price. Let, for example, the price of product X decrease. This leads to an increase in demand for this product. The increase in demand is due to two effects. Firstly, it is the increase in the consumer's real income. If the product is normal, this leads to an increase in demand for it. This effect is called the ***income effect***. Secondly, when the price of a product decreases, it becomes relatively cheaper than its substitutes. In view of this, consumer demand switches to this product. This is called the ***substitution effect***.

Based on the above theoretical aspects, it is intuitively clear that the substitution effect is opposite to the price change. The income effect is more complicated. If the product is normal, this effect is opposite to the change in its price. For low-quality goods, this effect is directly proportional to the price change.

There are two most popular methodological approaches to determining the effects of substitution and income. The first one is Slutsky's approach. Its graphical interpretation is shown in Fig.2.11.

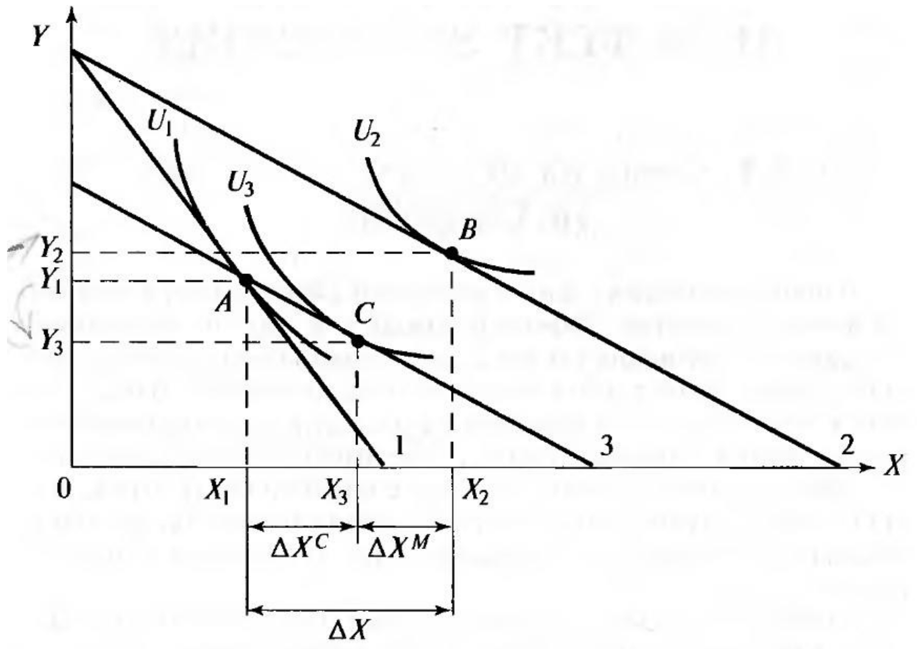


Fig.2.11. Graphical representation of the effects of substitution and income in Slutsky's interpretation.

Consider the graph shown in Figure 2.11. Budget constraint line 1 implies a constraint on the consumer's budgetary resources before the price of product *X* is reduced. After the reduction of the product price, the budget constraint line shifted to the right. In Figure 2.11, it is indicated by the number 2. The first budget constraint line touches the indifference curve U_1, the second one touches the indifference curve U_2.

At the old price of product *X*, the optimum is achieved at point *A*. Due to its decrease, the optimum shifts to point *B*. According to Slutsky's approach, the consumer's real income has not changed if he is able buy the same commodity set. Following this approach, Fig.2.11 shows the budget constraint line 3. It is parallel to line 2 and goes through point *A*. Line 3 touches the indifference curve at point U_3, determining the optimum at point *C*.

The difference between the demand for product *X* at point *B* and point *A* indicates the total income effect and the substitution effect. Meanwhile, the difference between the demand for product *X* at point *B* and point *C* indicates the income effect. And the difference between the demand for product *X* at point *C* and point *A* indicates the substitution effect.

As mentioned above, the substitution effect is always negative. In contrast, the income effect can be both positive and negative. Theoretical economists identify a special group of goods, the demand for which increases with the price of these goods. These economic goods are called ***Giffen goods***.

Actually, Giffen goods are rare. Typically, these are essential goods.

2.6 Market demand and elasticity coefficients

So far, we have considered the demand of an individual consumer for an individual product. However, on the market there are usually thousands or even tens of thousands of individuals who form the market demand for a certain product. The definition of market demand is as follows. ***Market demand*** is the sum of the functions of individual consumer demand in the market.

Thus, the definition of market demand can be represented as a formula

$$D_i(P_1, P_2, \dots, P_m, M) = \sum_{j=1}^{n} D_{ij}\left(P_1, P_2, \dots, P_m, M_j\right), \tag{2.14}$$

where D_{ij} is the demand of consumer *j* for product *i*; M_j is the income of consumer *j*; *n* is the number of consumers on the market; *m* is the quantity of substitute products.

Consequently, market demand depends not only on the total amount of demand of all consumers present in the market, but also on its distribution among these individuals.

Example 2.6. Let's consider two cases. Suppose in city *X* there are 10 thousand residents whose income is 30 thousand rubles for each person and 10 residents with incomes of 1 million rubles for each. At the same time, in city Y, there are 10 thousand people with incomes of 25 thousand rubles for each and 10 people with incomes of 6 million rubles for each. The total incomes of the population of these cities are identical – 310 million rubles. However, the demand of residents of these cities for food will be different. The demand for food in city *X* is likely to be higher than in city *Y*.

Meanwhile, typically, a seller has no accurate data on each customer's income. In this case, when conducting marketing research, salesmen most often focus on the dynamics of average income in their region.

Economic theory frequently uses the concept of elasticity index. In general, the elasticity of one variable in relation to the other shows relative change in the variable explained with a 1% increase in the variable explaining.

When analyzing market demand, three elasticity coefficients are usually used: price elasticity of demand, income elasticity of demand, and cross elasticity. Let's describe them in more detail.

Price elasticity of demand shows the percentage by which the demand for a product will change when the price of it changes by 1%. Price elasticity of demand is indicated as E_P^D. Another way, price elasticity of demand is expressed by the formula

$$E_P^D = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}. \tag{2.15}$$

Since demand almost always depends on the price negatively, price elasticity of demand is largely a negative value.

Price elasticity of demand is different at different points of the demand curve. Let's show this with an example. Let's assume that the demand is linear. The ways linear price elasticity of demand changes at different points is shown in Fig.2.12.

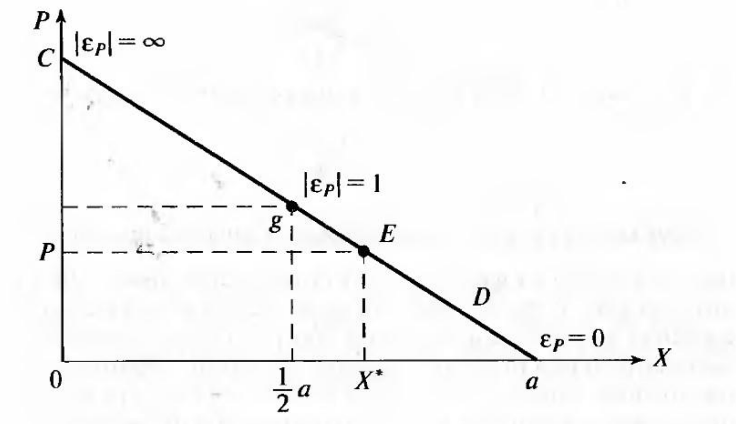


Fig.2.12. Price elasticity of demand at different points

The graph shown in Figure 2.12 illustrates that the higher the price of a product, the higher the elasticity of demand for this product. For example, a 1% reduction in the price of an expensive mobile phone can seriously affect the demand for it. At the same time, a 1% change in the price of a loaf of wheat bread is unlikely to have any serious impact on the demand for it.

Example 2.7. The product demand function is as follows: $Q^D = 10 - 2P$. Calculate price elasticity of demand at *P = 2.*

Solution. To calculate elasticity, let's use formula (2.15): $E_P^D = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}$. Thus, $E_P^D = -2\frac{4}{6} = \frac{-8}{6} = -1\frac{1}{3}$.

Answer: price elasticity of demand at *P = 2* is $-1\frac{1}{3}$.

The second most common measure of elasticity is income elasticity of demand. ***Income elasticity of demand*** shows the percentage by which the demand will change with an increase in consumer income by 1%. In contrast to price elasticity of demand, income elasticity of demand is not negative in most cases. Whether this measure is positive or negative depends on the type of product against which it is measured.

For normal goods, income elasticity of demand is positive. For low-quality goods, it is negative. Differential form of the formula for calculating income elasticity of demand:

$$E_M^D = \frac{\partial Q}{\partial M} \cdot \frac{M}{Q}, \tag{2.16}$$

where E_M^D is income elasticity of demand.

Finally, the third elasticity indicator is the cross elasticity of product *X* on product *Y*. ***Cross elasticity*** shows by how much percent the demand for product *X* will change when the price of product *Y* increases by 1%.

If goods X and Y are substitutes, the coefficient of cross elasticity is positive. It means that if one of the substitutes becomes more expensive, consumers will switch to the other.

In case goods X and Y are complementary, the coefficient of cross price elasticity of demand is negative: when the price of one of the complementary goods increases, the demand for it falls. Therefore, the demand for the second product from this pair will also fall because these goods can only be bought together.

The cross elasticity formula can be represented in differential form:

$$E_X^Y = \frac{\partial X}{\partial P_Y} \cdot \frac{P_Y}{X}, \tag{2.17}$$

where E_X^Y is the cross elasticity.

Thus, now we have considered economic and mathematical models of the consumer demand generation in the market. In the next chapter, we will consider the modeling of other market participants, which are manufacturers.

TASKS FOR SELF-SOLUTION FOR CHAPTER 2

1. Suppose that consumer preferences are described by the Cobb–Douglas utility function $U = \sqrt{XY}$. Derive the marginal substitution rate for this utility function.

2. Consumer preferences are represented by utility function $U = XY^2$. Find out the consumer's choice, if $P_X = 1$ ruble, $P_Y = 4$ rubles, and the consumer's income is 3000 rubles per month.

3. Consumer preferences are represented by utility function $U = \sqrt{X} + \sqrt{Y}$. Find out the consumer's choice, if $P_X = 3$ rubles, $P_Y = 1$ ruble, and the consumer's income is 120 rubles per month.

4. A student buys 8 units of product X and 4 units of product Y. The price of product X is equal to 2 monetary units. The marginal rate of substitution is 0.5. Find out the student's income.

5. A consumer with a utility function $U = XY$ has an income of 100 rubles. The price for product X is 5 rubles, the price for product Y is 1 ruble. The price of product X fell to 2 rubles. Find out the substitution effect and the income effect.

6. Demand function for product X is as follows $Q^d = 10 - 2P$. Find price elasticity of demand if price is 2.5. Give an economic interpretation to the value obtained.

7. The utility function of a consumer has a form $U = X^{2/3}Y^{1/3}$. Derive the demand functions for goods X and Y for these preferences.

SELF-CHECK TEST FOR CHAPTER 2

1. Simultaneous increase in both demand and supply will lead to:
    a) increasing price and sales;
    b) falling prices and increasing sales;
    c) falling prices and sales;
    d) price and sales effects will depend on the slope of the supply and demand curves.
2. What is the price elasticity of the horizontal demand curve equal to?
    a) zero;
    b) one;
    c) plus infinity;
    d) minus infinity;
3. Cross elasticity of demand for product *X* relative to the price of product *Y* is negative. This means that products *X* and *Y*:
    a) substitutes;
    b) complements;
    c) independent goods;
    d) Giffen goods.
4. An indifference curve shows:
    a) all commodity sets bringing similar utility to the consumer;
    b) the ratios of goods *X* and *Y* in which the saturation point is reached;
    c) the ratio of labor and capital at which the maximum level of utility is reached;
    d) the ratios of goods *X* and *Y* in which the same level of income is reached;
5. Commodity sets that bring the same utility to the consumer:
    a) lie on the same indifference curve;
    b) lie on the same demand curve;
    c) lie on the same supply curve;
    d) they lie on the same line of budget constraints.
6. At an optimum point, the slope of the indifference curve is equal to:
    a) the ratio of prices for goods *X* and *Y*;
    b) the ratio of the prices of resources;
    c) the ration of marginal utilities for goods *X* and *Y*;
    d) the ratio of the volume of goods *X* and *Y*.
7. When the consumer's income increases, the budget constraint line:
    a) will shift parallel up;
    b) will shift parallel down;
    c) will change its slope;
    d) will shift up, but not parallel.

8. Demand and price of complementary goods:
    a) change in the same direction;
    b) change in different directions;
    c) are independent from each other;
    d) all of the above is true.
9. Choose two products that are likely to be substitutes for most consumers:
    a) ballpoint pen and gel pen;
    b) paper and printer;
    c) boots and shoe polish;
    d) clock and refrigerator.
10. With an increase in income the demand for low-value goods:
    a) falls;
    b) increases;
    c) increases faster than income;
    d) can either increase or decrease.
11. Let's assume that the prices of all goods and the incomes of all consumers have increased by the same amount. In this case market demand for any product:
    a) will increase;
    b) will decrease;
    c) will remain unchanged;
    d) can either increase or decrease.
12. Assume that demand function for product X is as follows: $Q_X^D = 20 - P_X + 4M$. This means that product *X* is:
    a) a low-value product;
    b) a normal product;
    c) a luxury good;
    d) a Giffen good.
13. Let there be product *X*. Its income effect is greater than its substitution effect and acts in the opposite direction. It means that product *X* is:
    a) a normal product;
    b) a low-value product;
    c) a luxury good;
    d) a Giffen good.
14. With a 5% increase in consumer incomes, the demand for product *X* increased by 8%. It means that product *X* is:
    a) a normal product;
    b) a low-value product;
    c) a luxury good;
    d) a Giffen good.

CHAPTER 3. MATHEMATICAL MODELING OF CONSUMER BEHAVIOR

The previous chapter provided a brief overview of the theory of consumer behavior in the market. This chapter will examine the behavior of other market participants – manufacturers. It should be noted that here and further we assume that manufacturers are also sellers of their goods to end consumers. However, that is not always the case. For example, farmers produce agricultural products, but it is not them who most often sell these products to end consumers.

It is believed that a commercial company seeks to maximize its profits. Such company's offer is formed by maximizing profits.

The first half of the chapter describes the task a company sets in order to maximize its production. The second part describes the task of maximizing profits. These tasks are interconnected and lead to the mathematical theory of derivation of a firm's supply function.

3.1 Production function and its types

In the production process, a company uses certain resources which include financial, labor, property and others. As a formula it can be represented as follows:

$$Y = f(R_1, R_2, \dots, R_m), \qquad (3.1)$$

where Y is the company's output; R_i is the amount of resource *i* used; f is the production function.

Let's give a definition of the production function. ***Production function*** is the relationship between the maximum possible output of the company and the amount of resources it uses.

It should be noted that the production function expresses only the maximum possible total output. It does not reflect the output at which resources are not used efficiently enough.

In practice, a company hardly ever uses 100% of all the resources available. Firstly, the firm's production capacity is rarely utilized more than 80%. Secondly, not all people spend at work as much time as set by the employment contract. However, to build mathematical models, we assume that all factors of production are used as efficiently as possible.

To build visual economic and mathematical models and analyze them, all factors of production are arranged into two large groups - labor (indicated as L) and capital (indicated as K). It is also assumed that labor is divided into mental and physical.

So the production function looks as follows:

$$Y = f(K, L). \qquad (3.2)$$

The same volume of output can be produced with a different combination of labor and capital. Consider, for example, agricultural production of wheat. About 150 years ago in Russia, agriculture used a minimum amount of capital, which was compensated by a large number of people employed in agricultural work. However, after 50-70 years, the situation began to change.

The set of points in coordinates *K–L* representing the same output volume is called an ***isoquant***. In most cases, isoquants are hyperbolic. Typical isoquants are shown in Fig.3.1.

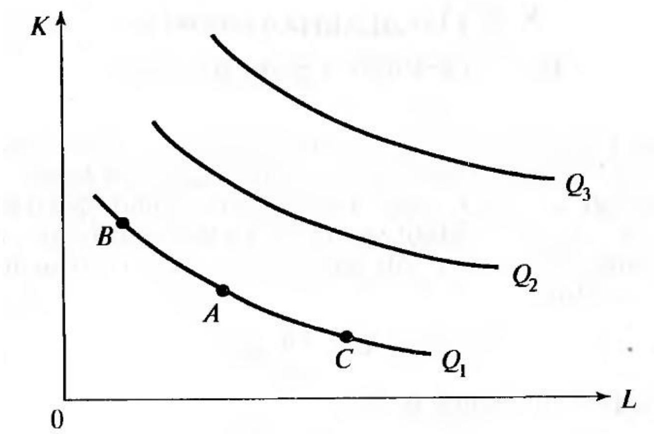


Fig.3.1. Graphic representation of isoquants

As can be seen from Fig.3.1, at points *A*, *B* and *C*, which represent different combination of labour

and capital, the firm reaches the same output, equal toQ_1.

The higher the isoquant is located, the greater output it displays. For example, in Fig.3.1 isoquant Q_2 represents a larger output than isoquantQ_1.

An important feature of the production function is its marginal rate of technological substitution (*MRTS*).

Marginal rate of technological substitution of capital by labor (represented as $MRTS_{K,L}$) shows how many units of labor should be abandoned with an increase in capital per unit so that the output volume remains unchanged.

$MRTS_{K,L}$ can be calculated through the marginal product of labor and the marginal product of capital. Before presenting this formula, let's consider definitions of these concepts.

Marginal product of labor (MP_L) shows how much output will increase with an increase in the amount of labor used per unit.

Mathematically, the marginal product of labor is a partial derivative of labour output

$$MP_L = \frac{\partial Y}{\partial L}. \qquad (3.3)$$

Marginal product of capital (MP_K) is determined in the same way, it is mathematically equal to the partial derivative of the capital output:

$$MP_K = \frac{\partial Y}{\partial K}. \qquad (3.4)$$

Marginal rate of technological substitution is equal to the ratio of the marginal product of labor and the marginal product of capital:

$$MRTS_{K,L} = \frac{MP_L}{MP_K}. \qquad (3.5)$$

Example 3.1. The production of a firm is characterized by a production function $Y = K^{0,3}L^{0,5}$. Find the rate of technological substitution of this production function. Calculate it at point (2; 3).

Solution. According to formula (3.3), the marginal product of labor is equal to the partial derivative of labor output$MP_L = \frac{\partial Y}{\partial L} = \frac{1K^{0,3}}{2L^{0,5}}$. Similarly, the marginal product of capital is determined:

$MP_K = \frac{\partial Y}{\partial K} = \frac{0,3L^{0,5}}{K^{0,7}}$.

Let 's calculate the marginal rate of technological substitution:

$MRTS_{K,L} = \frac{MP_L}{MP_K} = \frac{1K^{0,3}K^{0,7}}{2L^{0,5}0,3L^{0,5}} = \frac{K}{0,6L}$.

Next, we substitute values K and L into the resulting equation: $MRTS_{K,L}(2;\ 3) = -\frac{2}{0,6 \cdot 3} = 1,11$.

Answer: at point (2;3), the marginal rate of technological substitution is approximately equal to 1.11. Consequently, when capital increases by one unit, in order to keep the output unchanged, 1.11 units of labor must be abandoned.

Geometrically, value $MRTS_{K,L}$at a particular point can be interpreted as the tangent of the angle of inclination of the tangent line to the isoquant at this point.

Further the main types of production functions are considered.

The first type of production function is a function with interchangeable factors of production. Such a production function is called ***linear***. Graphically, isoquants of this function are straight lines (Fig.3.2).

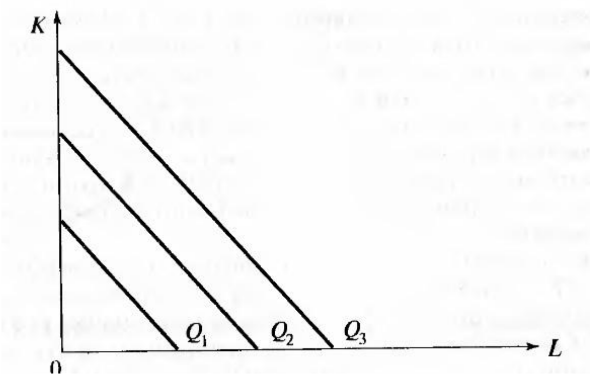


Fig.3.2. Isoquant map for linear production function

Mathematical representation of the linear production function is as follows

$$Y = aK + bL. \qquad (3.6)$$

It is easy to show that the *MRTS* of a linear production function is constant at any point. In other words, labor and capital always substitute each other in the same proportion. Such production processes are typical of early stage of industrialization. For example, one machine tool can substitute manual labor of five workers.

Now let's assume that factors of production do not substitute but complement each other in the same proportion. Such production processes are described by ***Leontief production function***. This type of a production function is named after the American economist of Russian origin, the Nobel Prize winner in Economics Wassily Leontief (1906-1999).

Isoquants of Leontief production function are broken lines (Fig.3.3).

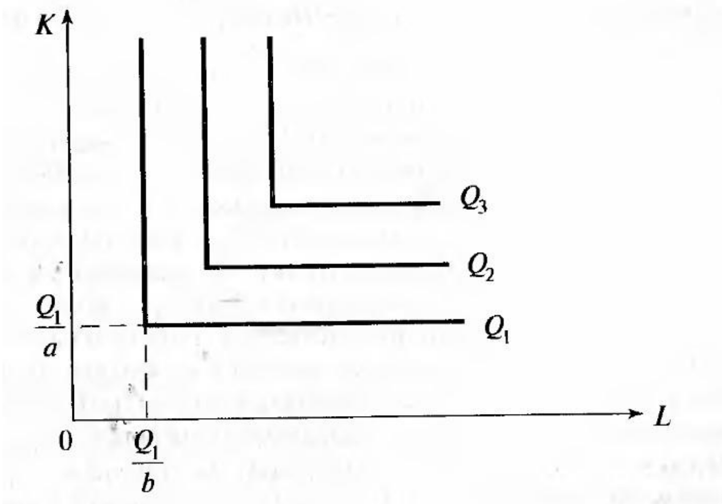


Fig.3.3. Isoquant map of the Leontief production function

Algebraic representation of production functions of this type

$$Y = min\{aK, bL\}. \qquad (3.7)$$

This type of production processes is typical for a mature industrial society. For example, in modern society, many automated processes can no longer be substituted by manual labor, as in the example with a linear production function. In this case the number of worker should be equal to the number of machine tools. Without a machine, the labor of the worker has no use. Thus, labor and capital complement each other.

However, the two considered examples of production functions describe extreme cases of a wide range of production processes in which labor and capital can partially and in different proportions substitute each other.

With the increase in the amount of labour, which compensates for the decrease in capital used, *MRTS* decreases. Hence, the isoquant is convex to the origin.

Such production processes are described by the ***Cobb–Douglas production function***.

The isoquant map of this production function has a standard form (Fig.3.4).

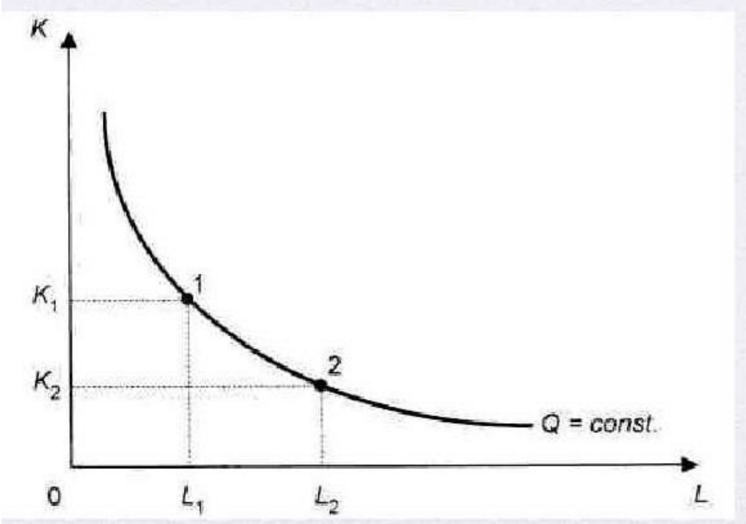


Fig.3.4. The isoquant of the Cobb – Douglas production function

MathematicallyFF, the Cobb – Douglas production function is as follows:

$$Y = AK^{\alpha}L^{\beta}, \tag{3.8}$$

where $A$is the degree of scientific and technological progress; α, β are parameters of the Cobb–Douglas function.

3.2 Properties of a production function. Scale effect and scientific and technological progress

Suppose a firm has increased the amount of labor and capital used by *N* times. So, its output should grow. Depending on the output growth, there are three types of production processes:

1) production processes with increasing returns to scale. In such processes, an output will increase by more than *N* times;
2) production processes with constant returns to scale. In such processes, an output will increase by *N* times exactly;
3) production processes with decreasing returns to scale. In such processes, an output will increase by less than *N* times;

Example 3.2. The production function of a firm is as follows $Y(K, L) = \sqrt{KL}$. What returns to scale is this production characterized by?

Solution. Let the firm increase the amount of resources used by n times. Hence $Y(nK, nL) = \sqrt{nK \cdot nL} = \sqrt{n^2}\sqrt{KL} = n\sqrt{KL} = nf(K, L)$. Consequently, there are constant returns to scale.

Answer: this production is characterized by constant returns to scale.

The concept of returns to scale can be interpreted through the average labour product or labor productivity. ***Average labour product*** (AP_L) shows an average number of production units produced by one unit of labor resources. Mathematically, the average product of labor is calculated by the following formula:

$$AP_L = \frac{Y}{L}. \quad (3.9)$$

If the average product of labor grows with the growth of output, there are positive returns to scale. If labor productivity decreases with the growth of production, there are negative returns to scale.

Example 3.3. Figure 3.5 shows a comparison of GDP dynamics and labor productivity in Russia for the period from 2012 to 2020. It is evident form the figure that in 2012-2016 Russian economy was characterized by constant returns to scale. And in 2017-2020, positive returns to scale were increasingly evident in the domestic economy.

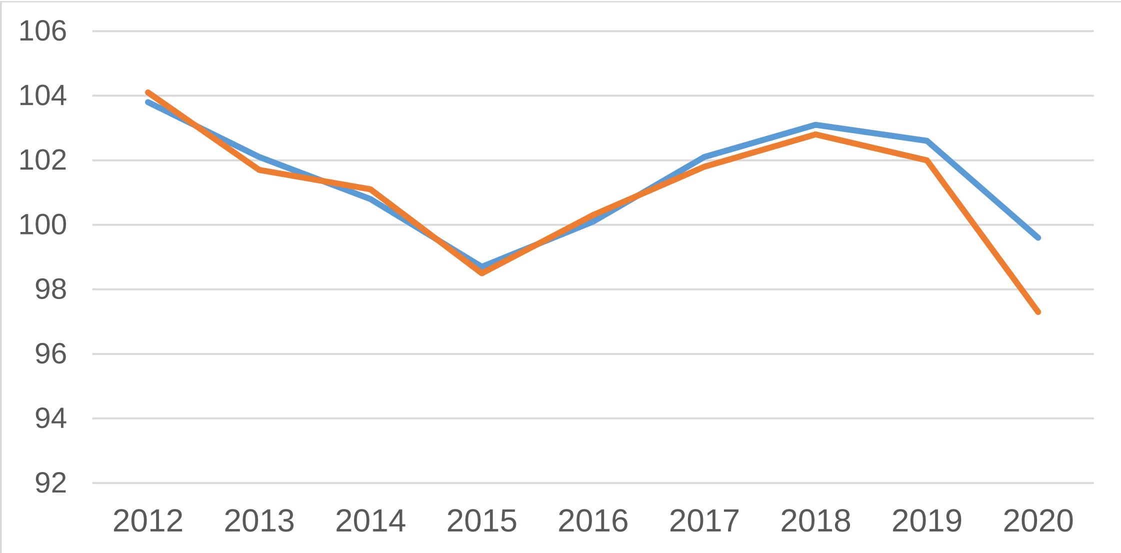


Fig.3.5. The comparison of GDP dynamics and labor productivity in Russia for the period from 2012 to 2020 (based on Rosstat publications)

An important feature of the production function is the ratio of capital to labour. ***Ratio of capital to labour*** shows the number of units of capital for one employed person.

The dynamics of the ratio of capital to labor determines the type of scientific and technological progress due to which the company's technologies develop.

In microeconomics, scientific and technological progress is the use of increasingly advanced technologies, staff development and improvement of existing business processes. The influence of scientific and technological progress makes the isoquant shift left and down (Fig.3.6). In this figure, solid lines indicate isoquants before scientific and technological progress, and dotted lines indicate isoquants after its impact.

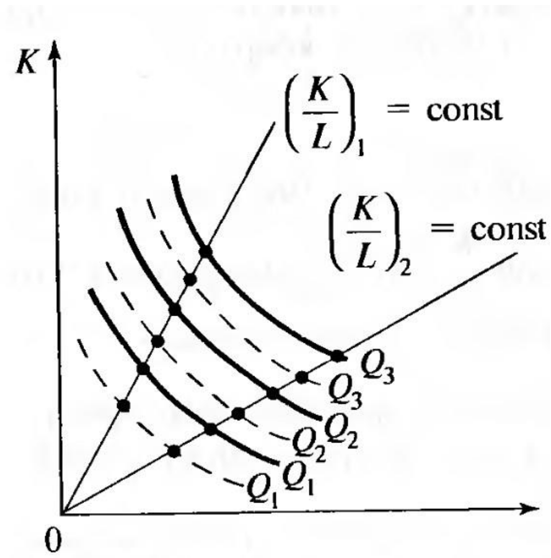


Fig.3.6. The impact of scientific and technological progress on the isoquant position

There are three types of scientific and technological progress: capital-intensive, labor-intensive and neutral.

Example 3.4. The production function of a firm is as follows $Y(K, L) = 4\sqrt{KL}$. Find average products of labor and capital at K = 16, and L = 4.

Solution. According to the formula (3.9), the average product of labor is $AP_L = \frac{Y}{L}$, and the average product of capital is determined by the ratio

$AP_K = \frac{Y}{K}$.

Then$AP_L = \frac{4\sqrt{KL}}{L} = 4\sqrt{\frac{K}{L}} = 8$,

$AP_K = \frac{4\sqrt{KL}}{K} = 4\sqrt{\frac{L}{K}} = 2$.

Answer: with the given amount labor and capital used, average output of one unit of labor is 8 units, and that of capital is 2 units on average.

3.3 Production costs and choice of manufacturer

In microeconomics are accounting and economic costs. Accounting costs are accrued in accordance with the current accounting legislation. Economic costs include also unofficial payments that are not accounted for in financial statements. Further, the term of costs will refer to accounting costs.

The difference between the revenue of a company's products and its costs is profit. Each firm reports its profits to the tax and statistical state authorities. Often firms tend to minimize the amount of costs with the existing volume of output. Another task is to maximize output at the current cost rate.

Let's assume that r is the rental rate per unit of capital. We also assume that the entire capital of the firm is leased; w is a wage rate. Thus, the total costs of the company can be represented as

$$TC = rK + wL, \tag{3.10}$$

where TC is company's costs.

Graphically, in coordinates K–L, the cost equation is a straight line. This straight line is called an ***isocost***. The isocost map is shown in Fig.3.7

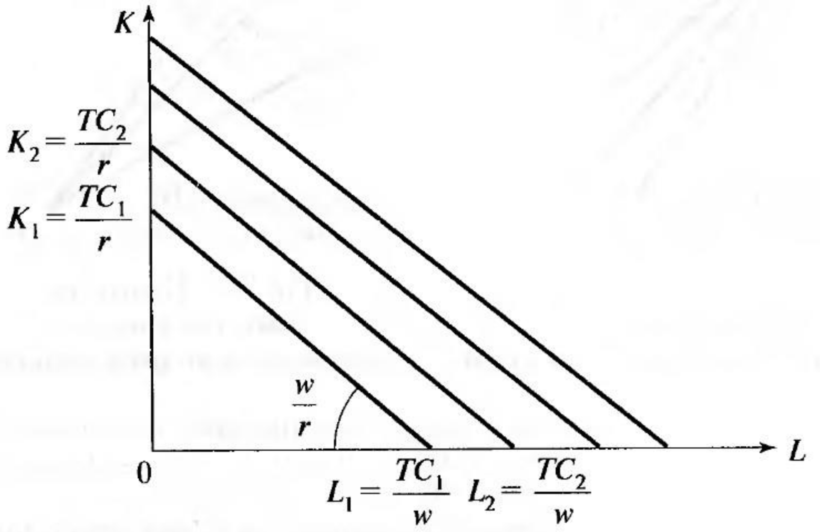


Fig.3.7. Isokost map

The slope of an isocost is determined by the ratio $\frac{w}{r}$.

Suppose a firm wants to maximize its output at a certain rate of financial resources. The question arises: what output volume will the firm choose? By the analogy with the consumer's choice, the manufacturer's optimum is achieved at the point where the isocost touches the isoquant.

The point where the isoquant touches the isoquant is called the ***manufacturer's optimum.*** Fig. 3.8 shows a graphical representation of the manufacturer's optimum. In this figure , the manufacturer 's optimum is reached at point *C* with the output volume Q_1. The firm will not produce goods in volume Q_1 as at an affordable costs it can achieve a higher output. At the same time, output Q_2 is unattainable for the company. The manufacturer's optimum is only achieved at point *C*.

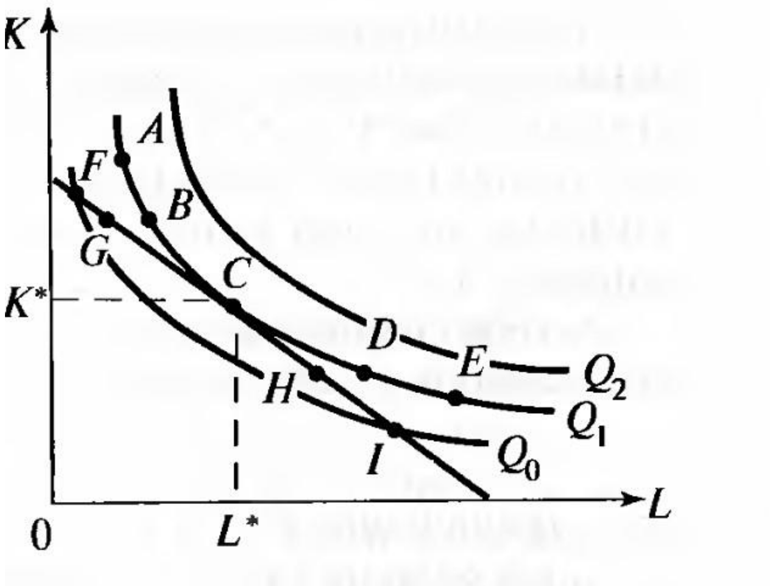


Fig.3.8. Graphic representation of the manufacturer's optimum

Mathematically, the manufacturer's optimum is achieved at the point where marginal rate of technological substitution is equal to the ratio of the prices of the resources used:

$$\frac{MP_L}{MP_K} = \frac{w}{r}. \tag{3.11}$$

In general, the task of choosing a manufacturer is as follows: a firm seeks to maximize its output at a given cost rate. Mathematically, this problem can be written as a conditional extremum:

$$\begin{gathered} Q = f(K, L) \to max, \\ wL + rK = C = const. \end{gathered} \tag{3.12}$$

Example 3.5. Some production technology is described by the following production function: $Q = 2K^{1/2}L^{1/3}$. The rate of pay is 10 rubles. The rental rate of the capital is 20 rubles. So, the company can allocate 600 rubles for production. What output is optimal for the company (in case of a fractional value, round up)?

Solution. According to the formula (2.9), the manufacturer's optimum is reached at the point where condition $\frac{MP_L}{MP_K} = \frac{w}{r}$ is satisfied. Let 's calculate the marginal products of labour and capital:

$MP_L = \frac{2K^{1/2}}{3L^{2/3}}$;

$MP_K = \frac{2L^{1/3}}{2K^{1/2}}$.

Taking this into account, let's rewrite the condition of consumer optimum for this task: $\frac{MP_L}{MP_K} = \frac{4K}{6L} = \frac{2K}{3L} = \frac{10}{20} = \frac{1}{2}$.

Or$\frac{2K}{3L} = \frac{1}{2}$.

Hence $3L = 4K$.

Next , let's write down the isocost equation $10L + 20K = 600$.

Taking into account the calculated relations $10L + 15L = 600$.

Or

$25L = 600;$

$L = 24;$

$K = 18.$

Based on this, we calculate the optimal output volume: $Q = 2 \cdot 18^{\frac{1}{2}} \cdot 24^{\frac{1}{3}} = 2 \cdot 4{,}24 \cdot 2{,}85 = 24$.

Answer. The optimal output of the company is 24 units.

3.4 Production costs

Microeconomics considers short-term and long-term periods. During the short term a firm can only change the amount of labor used. During the long term, a firm changes both the amount of labor and the amount of capital.

Here is an example of how production costs in the long term are determined by the production function of the firm and by the prices of resources.

Example 3.5. The technology of unloading railcars is described by the following production function: $Q = 10\sqrt{KL}$. The pay rate is 10 rubles. The rental cost of one unit of capital is 40 rubles. Find the dependence of the firm's costs on the output volume.

Solution. First, let's find the manufacturer optimum as a functional dependence of the output volume on the cost of resources. According to the formula (2.9), the manufacturer optimum is achieved at the point where condition $\frac{MP_L}{MP_K} = \frac{w}{r}$ is satisfied*.* Next, let's find the marginal products of labor and capital:

$MP_L = \frac{10K^{1/2}}{2L^{1/2}} = \frac{5K^{1/2}}{L^{1/2}},$

$MP_K = \frac{5L^{1/2}}{K^{1/2}},$

Hence, $\frac{5K}{5L} = \frac{K}{L} = \frac{1}{4}$.

Hence $4K = L$.

Next , let's write down the isocosta equation: $10L + 40K = C$

Or$20L = C, 80K = C$.

Hence $Q = 10\sqrt{\frac{C}{20} * \frac{C}{80}} = 10\frac{C}{40} = \frac{C}{4}$.

Hence, $C = 4Q$.

Answer. For the given production function and prices of resources , the dependence of costs on output is as follows $C = 4Q$.

In microeconomics there are fixed costs and variable, which together make up total costs. *TC* stands for total costs.

In the long term, costs depend on the output volume:

$$TC = TC(Q). \tag{3.13}$$

Variable costs are the costs that change with a change in the output volume. Variable costs usually include salaries of personnel involved in production; the costs of raw materials and materials. *VC* stands for variable costs.

Fixed costs are the costs that do not change with a change in the output volume. Fixed costs usually include the salaries of management and the costs of maintaining buildings. *FC* stands for fixed costs.

Fixed costs and variable costs make up total costs:

$$TC = FC + VC. \tag{3.14}$$

Fig. 3.9 shows graphic representation of the dependence of total, fixed and variable costs on the output volume. In this graph, function *FC* is represented by horizontal straight line because fixed costs do not depend on the output volume. The curves of total and variable costs are parallel.

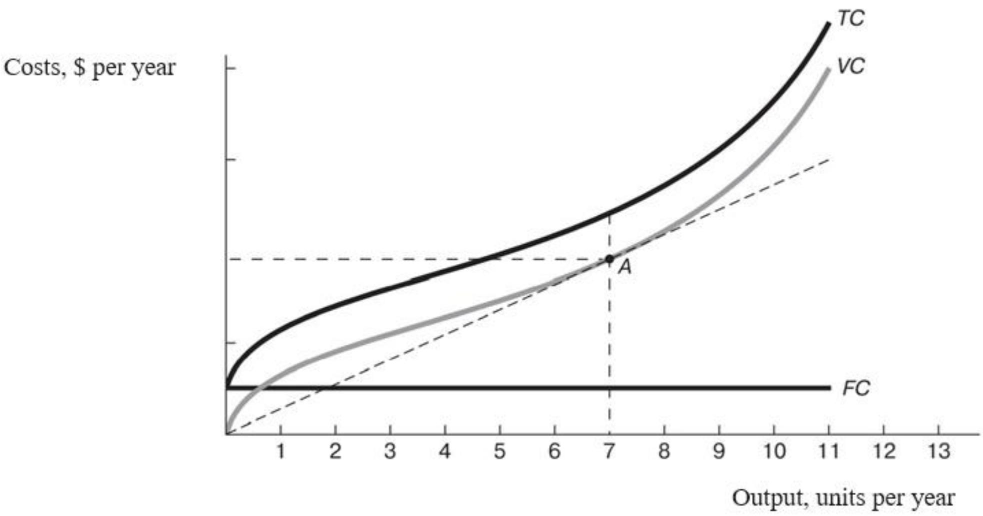


Fig.3.9. Dependence of total, variable and fixed costs on the output volume

In economic theory there are also marginal costs and average costs.

Marginal costs (*MC*) is the change in total production cost that comes from making or producing one additional unit. In terms of Mathematics, marginal costs are a partial derivative of the function of output total costs in terms:

$$MC = \frac{\partial TC}{\partial Q}. \tag{3.15}$$

Average costs (*AC*) show the average number of cost units are required to produce one unit of output. Average costs are calculated by dividing the total costs by the total output:

$$AC = \frac{TC}{Q}. \qquad (3.16)$$

Example 3.6. The production function of a company is presented by the ratio $Q = 2\sqrt{K} + +\sqrt{L}$. The pay rate is 10 rubles, the rental rate of a unit of capital is 15 rubles. Derive the dependence of the average and marginal costs of this company on the total output.

Solution. The problem can be solved similarly to example 3.5. The manufacturer's optimum achieved at the point where $\frac{MP_L}{MP_K} = \frac{w}{r}$. Let 's calculate the marginal products of labour and capital.

$MP_L == \frac{1}{2\sqrt{L}}$, .

$MP_K = \frac{1}{\sqrt{K}}$.

Hence, $\frac{\sqrt{K}}{2\sqrt{L}} = \frac{2}{3}$.

Hence $4\sqrt{L} = 3\sqrt{K}$.

Let 's square both parts of the equation $16L = 9K$.

Next , let's write down the isocosta equation:$10L + 15K = C$.

Taking into account earlier calculations $10L + \frac{16 \cdot 15}{9} L = C$,

$36\frac{2}{3} L = C$ or$65\frac{5}{27} K = C$.

Substitute the resulting ratios into the production function: Q.= $2\sqrt{\frac{3}{72} C} + \sqrt{\frac{27}{325} C} = 0{,}4\sqrt{C} + 0{,}29\sqrt{C} == \ 0{,}69\sqrt{C}$

Hence, $0{,}48C = Q^2$.

Hence $C = 2{,}08Q^2$.

Next, we find the functions of average and marginal costs . $AC = 2{,}08Q$,$MC = 4{,}16Q$.

***Answer*:**$AC = 2{,}08Q$,$MC = 4{,}16Q$.

It is assumed that the average cost curve is parabolic (Fig.3.10). As output increases positive returns to scale applies to the point of minimum. In this regard, the average costs are reduced. After that, negative return to scale applies and average costs increase.

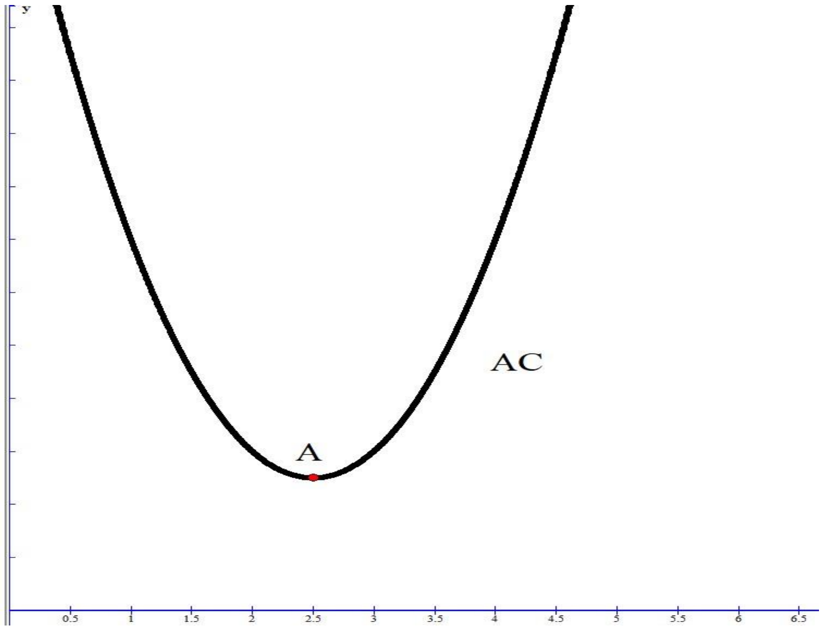


Fig.3.10. Average cost curve

Classical microeconomics also identifies average fixed and average variable costs. ***Average variable costs*** (*AVC*) show the average number of variable costs unit required to produce a unit of output. ***Average fixed costs*** (*AFC*) show the average number of fixed cost units required to produce a unit of output.

The graphs of the functions of average, average fixed, average variables and marginal costs are shown in Fig.3.11.

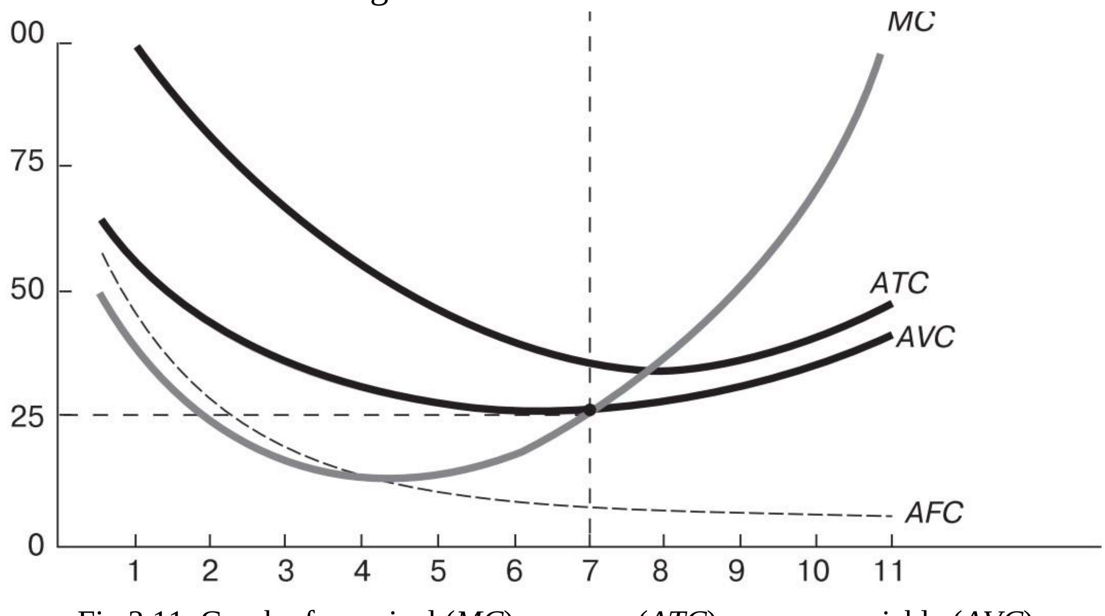


Fig.3.11. Graph of marginal (*MC*), average (*ATC*), average variable (*AVC*) and average fixed (*AFC*) costs

Since fixed costs do not depend on the total output, average variable costs decrease with the growth of Q as can be seen from Fig.3.11. Algebraically, average variable costs can be calculated by dividing the variable costs by the total output:

$$AVC = \frac{VC}{Q}. \quad (3.17)$$

Average fixed costs can be calculated in the same way:

$$AFC = \frac{FC}{Q}. \quad (3.18)$$

3.5 Profit maximization by a competitive firm and supply curve

According to classical microeconomics, the main goal of a firm is to maximize profits. However, in reality, this assumption is not always realized.

Firstly, non-profit organizations do not seek to maximize their profits. Religious organizations or political parties declare that their goals are not connected with profit maximization. Secondly, budgetary institutions often have other goals than profit maximization. For example, general education schools or public health institutions may make management decisions that lead to a decrease in their net profit. Thirdly, company owners are usually interested in maximizing profits, but its top management may not.

Further, we will consider the activities of enterprises interested solely in maximizing profits.

Profit is indicated by letter π. In our simplified model, profit is equal to the difference between the firm's revenue and its costs:

$$\pi = R - C. \quad (3.19)$$

Taking into account the fact that we consider only those firms that seek to maximize their profits, to find the total output that maximizes profit, let's find its derivative on *Q* and equate it to zero:

$$\frac{d\pi}{dQ} = MR - MC = 0, \quad (3.20)$$
$$MR = MC,$$

where *MR* is the ***marginal revenue*** of the company. Marginal revenue shows how much the company's revenue will change with an increase in its output by one unit.

Let's clarify the economic meaning of the equation (3.20). If the marginal revenue from one unit of goods exceeds the costs of its production, the firm can increase its profits by increasing the output. When an equilibrium between marginal income and marginal costs is reached, further increase in output is impractical.

In the market of perfect competition (for more details, see Chapter 4), the firm is a ***price-taker***. This means that a single firm cannot influence the market price. The organization only accepts the price that has been established in the market as a result of the interaction of supply and demand.

Thus, for a competitive firm, the price is a constant relative to the total output. Let's change the equation of a company's revenue, which is equal to the product of the price by the volume of sales: $R = PQ$. In this case, after differentiating the firm's profit function, we obtain the following condition for maximizing profit by a competitive firm:

$$P = MC. \qquad (3.21)$$

Example 3.7. The cost function of a firm operating in a market of perfect competition is as follows$C = \frac{Q^2}{2} - 2Q$. The market has established a market price equal to 2 monetary units per unit of production. Find the company's output.

Solution. According to (3.21), the condition for maximizing profit by a firm is the equality of price to marginal costs: $P = MC$. Let's calculate the marginal cost function: $MC(Q) = Q - 2$. Let's make marginal costs and the price equal: $Q - 2 = 2, Q = 4$.

Answer. The profit-maximizing firm will produce 4 units of production.

Consider the graphical representation of equation (3.21) (Fig.3.11). Let's assume that the firm is a price-taker and a certain price *P* has been established on the market. On the graph it will be represented by a straight line parallel to axis 0 *X* (see Fig.3.12).

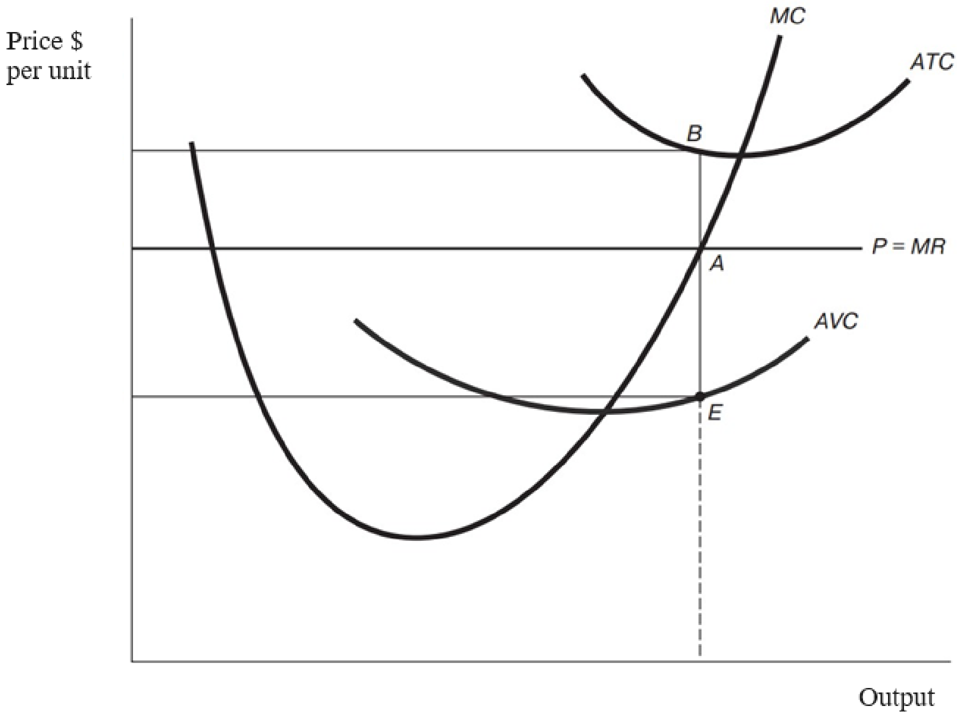


Fig.3.12. Graphic representation of the competitive firm's optimal output

The optimal output Q^* is set at the point where straight line P intersects with curve *MC*. In this regard, three cases can be considered:

1) $P > AC(Q^*)$. In this case the company makes a positive profit. It operates on the market successfully.

2) $AVC(Q^*) < P < AC(Q^*)$. When this ratio is met, the firm makes a negative profit but does not leave the market because if it does so, it will lose its assets, which may be illiquid.
3) $AVC(Q^*) > P$. If it is so the company leaves the market.

Thus, if the price set on the market is lower than min AVC then the company will definitely cease its operation. In this regard, the point where it is achieved min AVC,is a condition for the company to enter the market.

Example 3.8. The cost function of a firm operating in a market of perfect competition is as follows$C = Q^3 - 6Q^2 + 10Q + 8$. Find the output at which the company will enter the market.

Solution. As it was shown above, min AVC is the point of entry of the company into the market. Let's calculate the function of average variable costs: $AVC = \frac{VC}{Q}$;

$VC = Q^3 + 6Q^2 - 30Q;$

$AVC = Q^2 - 6Q + 30;$

$\frac{dAVC}{dQ} = 2Q - 6 = 0;$

$Q = 3.$

Answer: the company will enter the market when its output is 3.

The chapter reveales the features of economic and mathematical modeling of manufacturer behaviour. The next chapter describes firm behavior in different types of market structures.

TASKS FOR SELF-SOLVING FOR CHAPTER 3

1. The production of a firm is characterized by a production function $Y = K^{0,3}L^{0,5}$. Find the rate of technological substitution of this production function. Calculate it at point (2; 3).

2. The production of chairs is characterized by a production function $Q = K^{3/4}L^{1/4}$. The company's expenses amount to 250 rubles. The pay rate is 15 rubles. The capital expenditure rate is 11 rubles. Find the number of chairs that the company will produce (in the case of a fractional answer, round the value).

3. Solve the previous problem using the method of Lagrange multipliers. Find parameter λ. Give an economic interpretation of this parameter.

4. The production of shoes is characterized by the following production function: $Q = K^{1/4}L^{3/4}$. The pay rate is 8 rubles. The capital expenditure rate is 5 rubles. Derive the dependence of the firm's costs on the output.

5. The costs of a competitive firm are described by the formula: $TC = Q^3 + 6$. When the production stops, the costs are 4 monetary units. Find the output at which the company will enter the market.

6. The production function of the firm is as follows: $y = 2\sqrt{KL}$. Determine average products of labor and capital at $K = 9; L = 16$.

7. The costs of a competitive firm are described by the formula: $TC = Q^2 + 25$. Determine the maximum profit of the company at a price equal to 20 monetary units per unit of goods.

SELF-CHECK TEST FOR CHAPTER 3

1. The production function $y = f(K, L)$ is characterized by positive returns to scale. The amount of labor and capital used increases by 20%. In this case the output:

a) will increase by more than 20 %;
b) it will increase in the range from 15 to 20 %;
c) will remain unchanged;
d) it will increase by exactly 20%.

2. There is a production function $y = \sqrt{KL}$. It is characterized by:

a) positive returns to scale;
b) negative returns to scale;
c) constant returns to scale;
d) there is not enough data.

3. The pay rate is 10 rubles. Rental rate of capital is 20 rubles. In this case, the optimal output is achieved when the ratio of the marginal product of labor to the marginal product of capital is equal to:

a) 2;
b) 1,5;
c) 1;
d) 3.

4. A larger output is represented by isoquants that lie relative to the original one:

a) above and to the right;
b) below and to the right;
c) below and to the left;
d) above and to the left;

5. There is a production function:$y = K^{0,4}L^{0,3}$. According to this function, if the labor used increases by 1 %, the output volume will increase by:

a) 0,4 %;
b) 0,6 %;
c) 0,3 %;
d) 0,7 %.

6. Suppose that the firm's fixed costs have increased by the same amount by which variable costs have decreased. In this case, the total costs:

a) increased;
b) decreased;
c) did not change;
d) not enough information to answer.

7. In firm *A*, the average labor product is 10 with the number of permanent employees 15. In this case, the output of firm *A* is equal to:

a) 150;
b) 1,5;
c) 200;
d) 300.

8. Which of the following organizations do not have maximizing profits as their goal?

a) a general education school;
b) a political party;
c) a commercial bank;
d) an expensive restaurant owned by entrepreneur Petrov.

9. Factory *X* bought a production machine for 20 000 rubles and an office desk for the manager for 10 000 rubles. In this case, the variable costs will be:

a) 30 000 rubles;
b) 10 000 rubles;
c) 20 000 rubles;
d) 50 000 rubles;

10. The purchase of raw materials relates to:

a) to fixed costs;
b) variable costs;
c) marginal costs;
d) transaction costs.

11. Average costs are:

a) the sum of average fixed and average variable costs;
b) the sum of fixed and variable costs;
c) the sum of fixed and marginal costs;
d) the sum of fixed and average variable costs;

12. Let that there is a price in the market that is less than the average variable costs of firm *A*. In this case, the firm:

a) makes a positive profit and stays on the market;
b) makes a negative profit and remains on the market;
c) makes a negative profit and leaves the market;
d) makes a positive profit and leaves the market;

13. The company's cost function of the company is: $TC = 6q + 2q^2$. A company produces 25 units of goods and sells them for 36 rubles in a totally competitive market. In this case, the firm makes the profit of

a) -500 rubles;
b) 250 rubles;
c) -375 rubles;
d) 450 rubles;

14. The graph of the dependence of price on sales, which maximizes the seller's profit at given prices for resources is called:

a) the supply curve;
b) the demand curve;
c) an isoquant;
d) an isocost.

CHAPTER 4. MATHEMATICAL MODELING OF MARKET STRUCTURES

The chapter describe economic and mathematical models of a company functioning under the conditions of various types of market structures (types of market environment). Economic and mathematical models are the basis of an applied economic discipline - the theory of industrial markets.

The first section of the chapter provides an overview of the types of market structures. Further, special attention is paid to the types of market in which there are many buyers: perfect competition, oligopoly, monopoly and monopolistic competition.

4.1 Market structure and a market of perfect competition

What is a market structure? The type of market or market structure is determined by following features:

1) the number of sellers,
2) the number of buyers,
3) the nature of the product,
4) market entry conditions,
5) information mobility.

A market structure is a type of market that is characterized by the features mentioned above.

The first type of market is ***perfect competition***. These are markets with many sellers and many buyers. For this reason, none of the buyers and none of the sellers can influence the market price. Consequently, both sellers and buyers are price-takers.

In a market of perfect competition, all sellers sell their products at a single market price. Also, this type of market structure is characterized by the fact that there are no barriers to enter the market. Thus, there is fierce competition between sellers in the market. This leads to the fact that the profit of manufacturers is almost zero.

There are no barriers to enter the market of perfect competition. Consequently, if sellers in this market make a positive profit, new competitive manufacturers come to the market, reducing the profits of current producers.

Special attention should be paid to the uniqueness of market equilibrium in this type of market structure. There are goods whose supply is always higher than the demand for them. Thus, the equilibrium for such goods cannot be achieved. Such economic benefits are referred to as ***free***. Consumers can enjoy free benefits for free. Examples of free economic benefits are sea water, sunlight, etc.

Consider the impact of specific taxes on market equilibrium in a competitive market. For example, the government introduces a specific tax on product X. Examples of specific taxes are excise duty, VAT.

The analysis of the consequences of the introduction of specific taxes is presented in Fig.4.1.

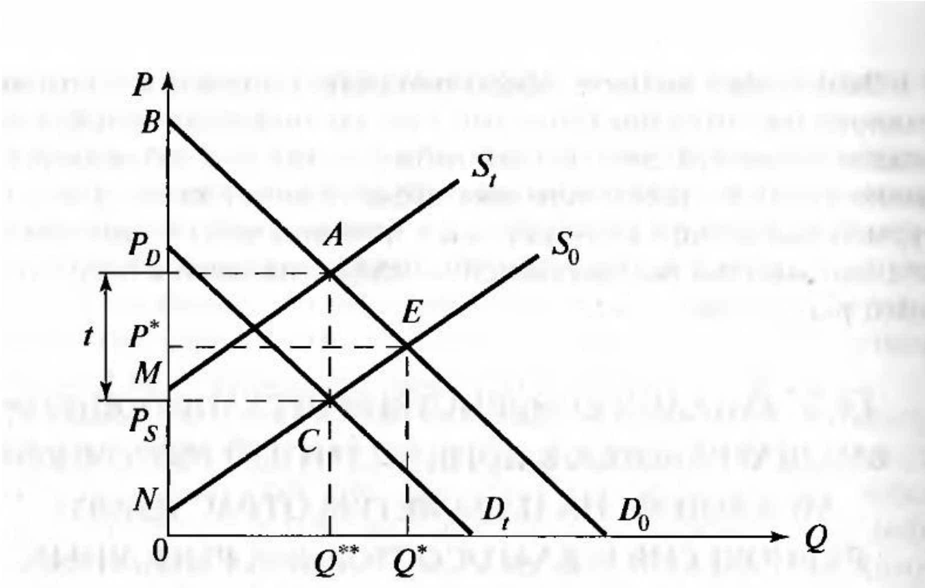


Fig.4.1. Analysis of the consequences of the introduction of a specific tax in the market of perfect competition

In Fig.4.1,D_0 is the demand curve. The line S_0indicates the demand in the market before the tax was introduced. Suppose that an excise tax was imposed on goods in the amount of *t* rubles per unit of goods. As a result, manufacturers will sell the same total of products at a price of *t* units higher. Therefore, the supply line is shifted up by *t* units. The new supply line $S_t$is shown in Fig. 4.1.

As can be seen from Figure 4.1, as a result of the introduction of a specific tax, the market price increases by *t* units. The equilibrium sales volume decreases from Q^*units to Q^{**}units. Thus, with the introduction of a specific tax on a product, its price increases, and the volume of sales falls.

Typically excise taxes are imposed on goods whose consumption is harmful to society. Common examples of excisable goods are alcohol and cigarettes. The Government imposes excise taxes on these goods to reduce their consumption and generate income from sales of excisable goods.

To sum up considering the market structure of a perfect competition, it must be noted that according to economic theory, there are a lot of sellers and a lot of buyers in a competitive market. Meanwhile, the question of the number of participants in the market for it to be recognized as competitive remains open. Therefore, this type of market relations is rather a general approximation of the process of setting prices for certain types of goods or services.

Example 4.1. The cost function of a firm operating in a market of perfect competition is described by the equation $C = \frac{Q^2}{20} + Q$. Derive the company's supply function.

Solution. The condition under which the company enters the market: $P > minAVC$. Since this equation has no constant, there are no fixed costs. Hence, $AVC = \frac{Q}{20} + 1$;

$\frac{dAVC}{dQ} = \frac{1}{20} = 0{,}05.$

$MC = \frac{Q}{10} + 1 = P$;

$Q = -10 + 10P.$

Answer. The equation of the supply line for this company is as follows $Q = -10 + 10P$.

4.2 Monopoly model

A monopoly is a type of market structure in which there is one seller and many buyers. In order for this type of market to arise in reality, there must be barriers to enter it. If there are no barriers, other producers will be able to enter the market, and it will no longer be a monopoly.

All barriers to enter to the market can be divided into legal and technological. Legal barriers include the legally enforceable right to produce certain goods. These may be patents, licenses, and similar legal rights. For example, in the Russian Empire, for a long time, the state owned a monopoly on alcohol production, since it was legally enshrined.

Technological barriers occur when the technology of production of a product or service is not publicly available. For example, the United States for a long time had a monopoly on the production of super-heavy missiles.

While in the market of perfect competition a firm is a price-taker, the monopoly has to decide on the size of the optimal price. There is an opinion that the monopolist is not limited in its pricing. In fact, this is not quite true. A monopolist cannot set a price for a certain quantity of goods higher than consumers are willing to pay for it.

Let's derive a formula for establishing equilibrium in the monopoly market. Thus, according to the formula (3.19), a firm's profit is the difference between its revenue and costs, the first derivative of profit $\frac{d\pi}{dQ} = MR - MC = 0$. Hence, the monopolist equilibrium condition

$$MR = MC. \tag{4.1}$$

A firm has monopoly power if it can influence the price that has been established in the market. Consequently, in any non-competitive market there is both a demand for the products of all manufacturers and a demand for the products of one particular firm.

Although the products of different competing firms producing the same product (for example, passenger cars) are often quite homogeneous, each market participant produces goods having specific consumer qualities. Due to this, in a monopolized market, different manufacturers can sell their products at different prices. In the case of a monopoly, the demand for the company's products corresponds to the market demand.

Let's change the company's revenue function: $R = P(Q) \cdot Q$. and find the derivative of revenue by Q. In this case , you need to find the derivative of the product $MR = \frac{dP}{dQ}Q + P(Q) == P(1 + \frac{1}{E_D^p})$. Given that at the monopolist's optimum point, its marginal revenue is equal to marginal costs, we obtain another expression for the monopoly equilibrium:

$$P = MC(1 + \frac{1}{E_D^p})^{-1}. \quad (4.2)$$

The pricing scheme described by formula (4.2) is called "costs+".

Let's graphically represent the monopolists's optimal choice (Fig.4.2). In Graph 4.2, the demand for the company's products is represented by line *D*. Line *MR* corresponds to the monopolist's marginal revenue. Line *SMC* line represents marginal costs.

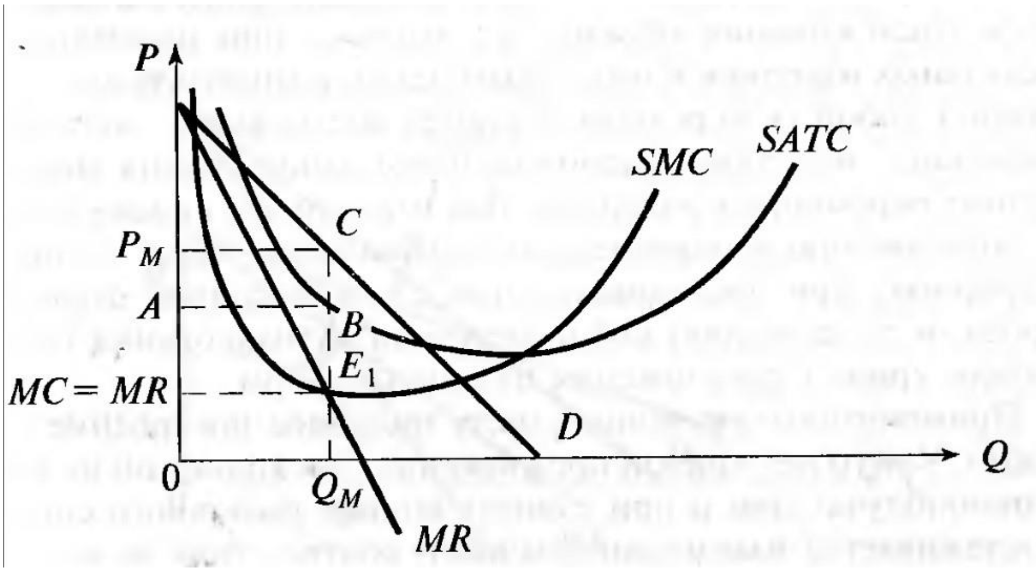


Fig.4.2. Monopolist's optimal choice

Monopolists's optimum is reached at point E_1, where marginal costs are equal to marginal revenue. In Figure 4.2, the optimal output is Q_M. Theoretically, a monopolist can set the price at the level at which marginal revenue and marginal costs are equal. However, with the appropriate total of output, the monopolist can raise the price to the level of demand. Therefore, the optimal price that maximizes the monopolist's profit is the price P_M shown at point *C* on line *D*.

Example 4.2. A monopolist firm has a cost function$TC = 20Q + 640$. Find the optimum of this firm for the demand function $Q = 50 - 0,5P$.

Solution. According to the formula (4.2), the monopolists's optimum is achieved at the point where $MR = MC$. Calculate the marginal costs: $MC = 20$. The company's revenue is equal to the product of sales price by sales volume

$TR = QP = (100 - 2Q)Q = 100Q - 2Q^2$.

Hence, $MR = 100 - 4Q$.

Then the monopolist's optimum is found by the ratio

$100 - 4Q = 20$;

$4Q = 80$;

$Q = 20$.

To determine the optimal price, we put the optimal volume into the demand function: $P = 100 - 40 = 60$.

Answer. Optimal sales volume for the company: 20 units at a price of 60.

It can be shown that the monopolist has a smaller output than the one that could be with the same demand if there were several manufacturers on the market. At the same time, the monopolist sets a higher price than the one that develops with the competitive interaction of several sellers.

In this regard, there is an issue of assessing the damage that a monopoly brings to society. Suppose that a monopolist's marginal costs are constant. The monopolist's optimum for this case is shown in Fig.4.3.

If the firm acted as a price-taker, it would choose the total of outputQ_C. However, as a result of the emergence of a monopoly in the market, the sale volume Q_M is set, which is less than Q_C, and the price P_M is set, which is higher than the competitive priceP_C.

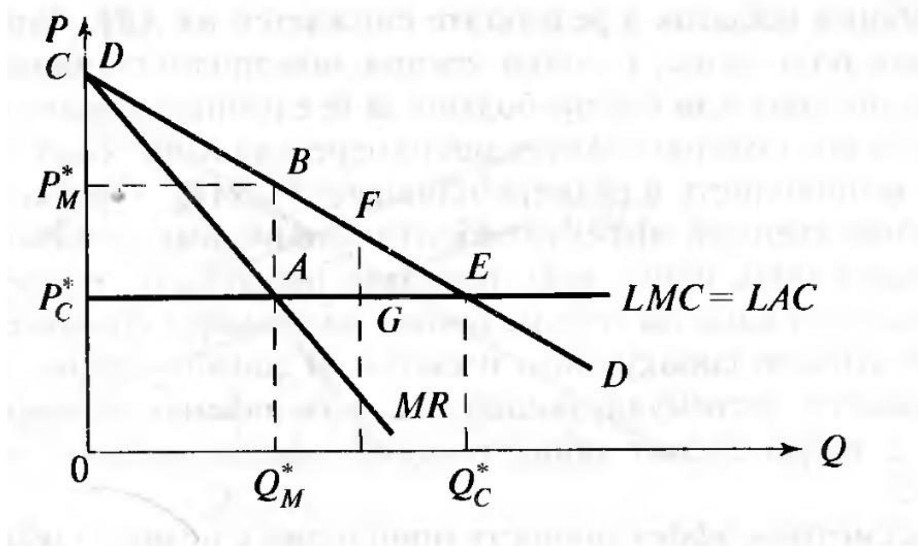


Fig.4.3. Graphic representation of the social costs of a monopoly

These considerations show that in case of monopoly the social surplus is reduced by an amount equal to the area of triangle *ABE.*

One of the common types of monopolies is so-called ***natural monopoly***. Natural monopoly refers to a situation where the existence of more than one producer on the market is economically impractical. A natural monopoly can satisfy the demand for its products at lower costs than the total costs would be if there were two or more producers.

This situation occurs when the demand line intersects the average cost curve before the minimum point of function *AC* is reached. This is typical for firms that have high fixed costs and relatively low variable costs (such companies include railway carriers, housing and utilities companies, etc).

The optimum of a natural monopoly is shown in Fig.4.4.

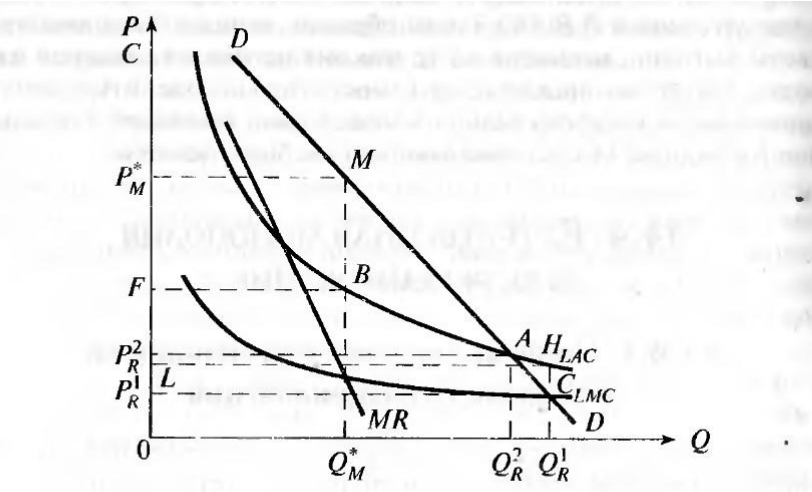


Fig.4.4. Graphic representation of the optimum of a natural monopoly

As can be seen from Figure 4.4, if a firm sets a price corresponding to the intersection of curve *MC* and the demand line, which would mean equilibrium in a competitive market, its profit will be negative. If a monopolist sets the price based on the total of output at which *MR* = *MC*, its profit is positive.

Example 4.3. The demand for telephone services in city *N* is set by the function $Q = 1000 - 50P$. The cost function of a monopolist firm is $TC = 500\ln(0{,}1Q - 20)$at *Q* > *200*. Is this monopolist firm a natural monopoly?

Solution. As mentioned above, a firm is a natural monopoly if its output at which curve *MC* intersects the demand line, brings a negative profit. Let's find function *MC*: $MC = \frac{dTC}{dQ} = \frac{50}{0{,}1Q-20}$.

Let's express the demand function by Q:$P = 20 - 0{,}05Q$.

Let's make the functions of marginal costs and demand equal: $\frac{50}{0{,}1Q-20} = 20 - 0{,}05Q$;$50 = (0{,}1Q - 20)(20 - 0{,}05Q) = 2Q - 0{,}005Q^2 - 400 + Q$;

$0{,}005Q^2 - 3Q + 450 = 0$;

$D = 9 - 9 = 0$;

$Q = \frac{3}{0{,}01} = 300.$

Next, we find the profit at this point:$TR = PQ = 20Q - 0{,}05Q^2$;

$\pi = TR - TC = 20 \cdot 300 - 0{,}05 \cdot 300^2 - 500 \cdot ln10 = 6000 - 4500 - 1151{,}3 = 348{,}7.$

Answer. This company is not a natural monopoly.

In all the models discussed above, the monopolist sets a single price for its products. Meanwhile, in an uncompetitive market, a firm may realize price discrimination. ***Price discrimination*** refers to a situation when for different buyers the firm sets different prices for identical units of its products.

In order for price discrimination to exist in the market, it is necessary that buyers do not have an opportunity to resell (arbitrage). If arbitrage is possible, consumers who buy products at a lower price can sell them to those to whom the manufacturer offers the same products at a higher price.

The purpose of price discrimination is to increase the seller's profit by reducing the consumer's surplus.

Price discrimination of the first degree occurs when the manufacturer sets an individual price for each buyer according to their willingness to pay. For example, an IT company has developed a new software product and is planning to sell it to five potential customers. This software manufacturer knows the economic effect of the implementation of its product. In this case, the IT firm will set a price for each buyer which will be equal to the economic effect of the implementation of the purchased software, and thereby it will be price discrimination.

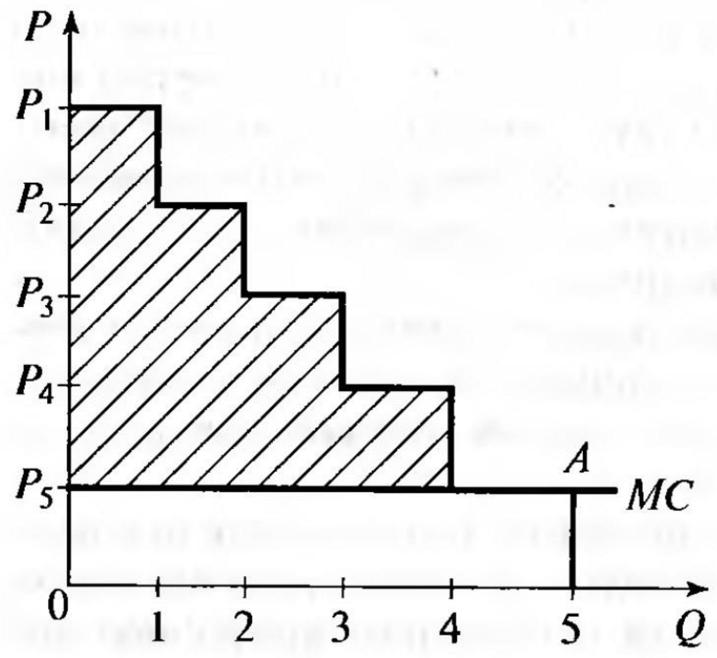


Fig.4.5. The case of price discrimination of the first degree

A graphical representation of the described situation is given in Figure 4.5. If a manufacturer set a price equal to the price under perfect competition, there would be the cost P_5per unit of on the market. In this case, the IT company would make a profit equal to the area of the rectangleP_5A50. However , when implementing price discrimination of the first degree , the seller's revenue is equal to the area of stepped figure $05P_1$.

Thus, in the case of price discrimination of the first degree, consumer's surplus is converted into seller's revenue. In this case, the demand curve also becomes the monopolist's marginal revenue curve.

Example 4.4. The design bureau has designed a new technology for the production of cars. The design costs were 500 rubles. The cost of technology transfer to one customer is 100 rubles. Five potential buyers are ready to buy this technology. The maximum price the first customer is ready to pay for the technology is 200 rubles, the second customer – 250 rubles, the third – 220 rubles, the fourth – 280 rubles, the fifth customer – 310 rubles. The manufacturer is aware of these prices. What profit will the design bureau make if it sells its technology to each buyer at an individual price? How much will this profit be higher than if the technology was sold to all buyers at a single price of a competitive market?

Solution. If the design bureau sells its technology at the price of a competitive market, then condition $P = MC$ must be met. The general cost function has the form: $TC = 500 + 100Q$; $MC = 100 = P$. Consequently, the company will sell the technology to all five customers at a price of 100. $\pi = R - TC = 100Q - 500 - 100Q = -500$. Thus, when selling products at a single price, the company will incur a loss of 500 rubles. In case the company sells the technology at different prices $R = 200 + 250 + 220 + 280 + 310 = 1260$. $\pi = 1260 - 500 - 500 = 260$. Thus, if the company sells products at an individual for each buyer price, its profit will be 260 rubles, which is 760 rubles more than in the case of a single price.

Answer. Thus, if the company sells products at an individual for each buyer price, its profit will be 260 rubles, which is 760 rubles more than in the case of a single price.

However, in practice, firms do not have information about the maximum price which a consumer is ready to pay for a certain amount of goods. Therefore, in reality, price discrimination of the first degree is extremely rare.

Price discrimination of the third degree has become more widespread, when prices are set not for each individual consumer, but for groups of consumers. In marketing, this pricing method is called ***market segmentation.***

An example of price discrimination of the third degree is different cost of visiting a museum for citizens of the country and foreigners.

Suppose some museum sets different prices for foreign visitors and visitors having citizenship of the country. The demand for the museum 's services among foreign citizens is D_1, the one among domestic tourists is D_2. The total demand is D_{Σ}. Assume that the marginal costs of servicing both domestic and foreign visitors are the same. A graphical representation of the monopolist's optimum in the case of price discrimination of the third degree is shown in Fig.4.6.

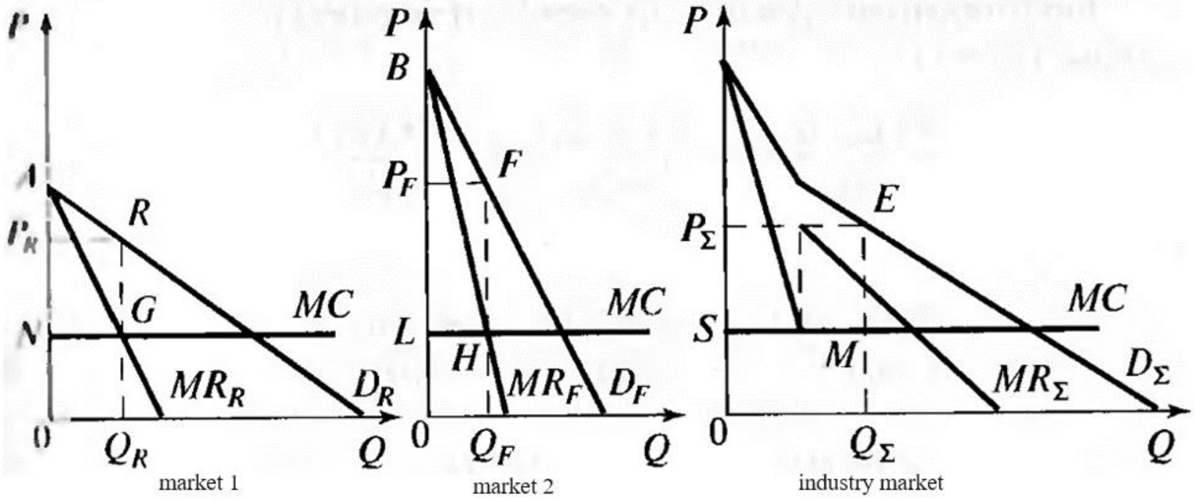


Fig.4.6. The choice of a monopolist in the case of price discrimination of the third degree

As it was shown earlier, the condition under which the monopolist's optimum is reached is the relation$MR = MC$. If a firm implements price discrimination of the third degree, this condition must be met in each of the market segments in which the seller works.

Figure 4.6 b shows that a monopolist who does not apply price discrimination would set the priceP_Σ. However, the seller's monopolistic behavior allowed raising the price in the second segment, thereby reducing the consumer surplus.

Based on the above, we can formulate a rule for maximizing profits under price discrimination of the third degree: the marginal profit in each of the market segments should be equal to the total marginal costs. The algebraic form of this condition is as follows:

$$MR_1 = MR_2 = \ldots = MR_n = MC_\Sigma. \tag{4.3}$$

Let's consider the case when market is divided into two segments. Since according to (4.2) $MR = P(1 + \frac{1}{E_P^D})$, the condition (4.3) can be represented in a different way:

$$P_1\left(1 + \frac{1}{E_P^D}\right) = P_2\left(1 + \frac{1}{E_P^D}\right). \tag{4.4}$$

It follows from condition (4.4) that if $E_{1,P}^D > E_{2,P}^D$, then $P_1 < P_2$. In other words, the more elastic the demand in a market segment, the lower price set by the seller is this segment. This conclusion seems quite logical in terms of the situation when a museum sets different prices for citizens of the country and foreign tourists. The demand of foreign tourists for the museum's services is less price-sensitive, since they incur large expenses for travelling to the city where the museum is located. Consequently, the elasticity of demand of foreign citizens for the price is lower, and the museum can set a higher price for its services for them.

Example 4.5. The demand for product *X* in Moscow is described by the equation $P_1 = 20 - Q_1$. In St. Petersburg, the demand for the same product is described by the formula $P_2 = 10 - Q_2$. The production costs of this product can be described by the equation: $TC = 5 + 0{,}5Q$. Calculate the company's profit under price discrimination of the third degree.

Solution. The company's costs function in market segmentation:

$TC = 5 + 0{,}5(Q_1 + Q_2)$.

Hence $MC_1 = 0{,}5Q_1$;

$MC_2 = 0{,}5Q_2$;

$R_1 = (20 - Q_1)Q_1 = 20Q_1 - Q_1^2$;

$MR_1 = 20 - 2Q_1$; $MR_2 = 10 - 2Q_2$.

Equate marginal income in Moscow with marginal costs:

$20 - 2Q_1 = 0{,}5Q_1$;

$2{,}5Q_1 = 20$;

$Q_1 = 8$.

Equate marginal income in St. Petersburg with marginal costs: $10 - Q_2 = 0{,}5Q_2$;

$Q_2 = 6{,}66$.

From this, we calculate the prices for product *X* in Moscow and St. Petersburg:$P_1 = 12$;

$P_2 = 3{,}33$.

$\pi_1 = 8 \cdot 12 - 5 - 4 = 87$.

$\pi_2 = 3{,}33 \cdot 6{,}33 - 5 - 1{,}68 = 15{,}5$.

Total total profit: $\pi = \pi_1 + \pi_2 = 102{,}5$.

Answer. The profit of a firm under third-degree price discrimination is 102.5 monetary units.

The last type of monopoly behavior is ***second-degree degree price discrimination***. With discrimination of this type, the seller has no opportunity for market segmentation. However, the seller sets certain conditions under which price reduction is possible. The examples of the second-degree price discrimination are discounts when passing the threshold corresponding to a certain sales volume, or price discrimination by time.

A typical example of the second-degree price discrimination is the sale of electricity at different rates day and night. This practice is practical because the demand for electricity at night is less sensitive to price changes.

A special type of the second-degree price discrimination is the so-called double-tariff pricing. In this case, the seller sets a fixed price for the right to purchase a service in the amount of *A*. Also , for the purchase of each unit of product the buyer pays price $\bar{P}$. Thus, the total cost of purchasing *Q* units of goods is

$$P = A + \bar{P}Q. \qquad (4.5)$$

4.3 Basic models of monopolistic competition

Monopolistic competition is a type of market structure where there are large numbers of both buyers and sellers. However, the goods of different manufacturers are differentiated. In other words, the consumer pays attention to which manufacturer's product they purchase. If in the market of perfect competition different manufacturers are supposed to produce almost similar products (this is possible, for example, in the market of crude oil or grain), then in the case of monopolistic competition, the products of different manufacturers differ in their consumer properties (for example, cars or computers).

The first and simplest model of monopolistic competition is the model of E.H. Chamberlin. This model is based on the concept of a product group. A product group refers to goods produced by different manufacturers which have similar consumer qualities and are partial substitutes.

The demand curve for the company's products has a negative slope, since if the price increases, the company will lose customers, but not all. Unique characteristics of a product produced by a seller can attract regular customers who prefer the products of this particular manufacturer to similar products of competitors.

The model of E. Chamberlin has two main notions. First, according to this model, the cost curves of different producers are identical. Secondly, if a firm reduces the price for its products, it deprives its competitors of their regular customers.

In monopolistic competition market structure, the demand is divided into the demand for the products of the entire industry and the demand for the products of an individual manufacturer in this industry. The demand for the products of an individual manufacturer is indicated by *d*. The demand for the company's products, if they did not have unique consumer qualities, is indicated by *D*.

A graphical analysis of the equilibrium of a firm in the market of monopolistic competition is presented in Fig. 4.7.

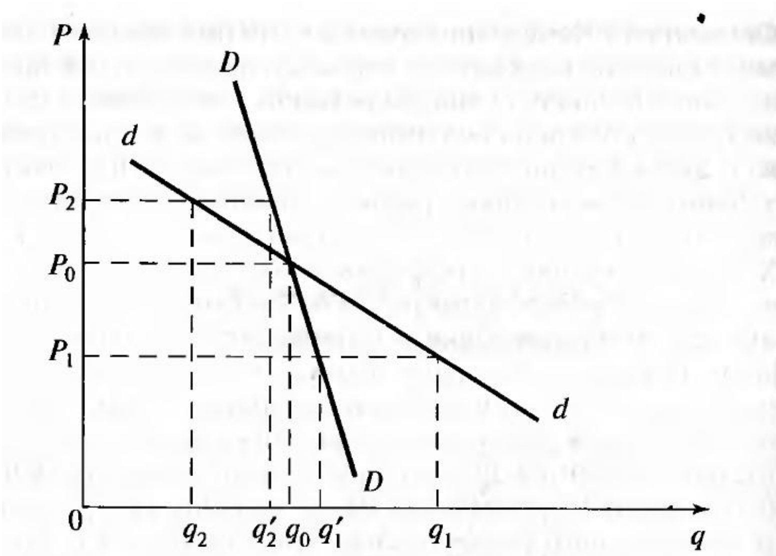


Fig.4.7. Equilibrium in monopolistic competition

As can be seen from Fig.4.7, curve *d* is less elastic than curve *D*. This is due to the fact that in the market of perfect competition, a firm can be a monopoly due to the differentiation of products from different manufacturers.

Suppose a firm lowered the price for its products from $P_0$to P_1. This means that in the short term , it will be able to increase sales from $q_0$to q_1. However, its competitors will also have to lower the prices for their products in order to get some of regular customers, who had left, back. Because of this, the company's sales volume will reduced to q_1'. Therefore, the firm's equilibrium point in the market of monopolistic competition is the point where the lines *D* and *d* intersect.

In monopolistic competition, a firm makes the same profit as in the market of perfect competition. However, it sets higher prices and its total output is smaller.

The product's spatial differentiation model is a special case of monopolistic competition model. According to this, different manufacturers produce relatively identical products. But they are different by their territorial location and proximity to their consumers. Obviously, the producers located closer to the consumers of products have lower transportation costs and, at the same market price, can make larger profits. This model was proposed in the late 1920s by H. Hotelling.

Suppose there are two ice cream stalls on the beach. The price in the first stall is P_A, in the second one it is P_b. The opportunity costs of ice cream transporting are *c* per unit distance. Then the real price of ice cream in stall *A* is:

$$P = P_A + cl, \quad (4.6)$$

where l is the distance between the stall and the consumer.

Next, let *A* be the distance from the right edge to the first stall, and *B* be the distance from the second stall to the left edge. *X* is the market share of the first stall. *Y* is the market share of the second stall. So it can be shown that

$$X = \frac{1}{2}\left(L - a - b + \frac{P_A - P_B}{c}\right). \quad (4.7)$$

Example 4.6. Suppose, on a city beach which is 100 m long, there are two ice cream stalls. The stalls are located 40m and 10m from the left and right ends of the beach, respectively. The production costs of ice cream are 15 rubles. Consumers are willing to pay 0.1 rubles for each meter of transportation of ice cream to their sunbeds. There are 100 people who want to buy ice cream on the beach. They are equally spaced form the stalls. Find the profit.

Solution. In this case

$P_A = P_B = 15;$

c = 0,1;

L = 100;

A = 40;

B = 10.

Hence, according to the formula (4.7) we get $x = \frac{1}{2}(100 - 40 - 10 + 0) = 25$;
$y = 100 - 40 - 10 - 25 = 25$;
$P = 15 + 2{,}5 = 17{,}5$;
$Q_1 = 25 + 40 = 65$;
$Q_2 = 25 + 10 = 35$;
$\pi_1 = 65 \cdot (17{,}5 - 15) = 162{,}5$;
$\pi_2 = 35 \cdot (17{,}5 - 15) = 87{,}5$.

Answer. The first stall will make 162.5 monetary units of profit, the second stall – 87.5 monetary units.

4.4 Basic oligopoly models

An oligopoly model is characterized by a relatively small number of sellers competing with each other. Despite the competition, different manufacturers produce homogeneous products.

Currently, there are several oligopolistic market models, which are generally accepted. The most famous of them are the Cournot model, the Stackelberg model and the Bertrand price wars model. Operating in the oligopoly market, the firm cannot ignore its competitors.

Suppose that in some industry there are *n* firms with different functions of total costs TC_i. Consequently, each firm has its own marginal cost function. The total output of the industry is Q. An individual company's output is q_i. Each firm maximizes its profit. The profit function of the *i*-th company is:

$$\pi_i = P(Q)q_i - TC_i, \tag{4.8}$$

where $\pi_i$is the profit of the *i*-th firm.

The firm seeks to sell a volume of products that would maximize its profit. Therefore, it is necessary to find the partial derivative of the profit function by the total output of the *i*-th firm: $\frac{\partial \pi_i}{\partial q_i} = \frac{\partial P}{\partial q_i} q_i + P(Q) - MC_i = 0$. Assume also that the demand for the industry goods is linear: $P(Q) = a - bQ$. Hence $\frac{\partial P}{\partial q_i} = -b$. In this case , the partial derivative of the profit function can be presented in a different form $-bq_i + P(Q) - MC_i = 0$. Therefore, the condition for maximizing profit by the *i*-th firm is the meeting of the ratio

$$q_i = \frac{P - MC_i}{b}. \tag{4.9}$$

Thus, as follows from equation (4.9), the firms that, due to technological advantages, have lower costs per unit of production have a larger market share in terms of sales.

Now, for simplicity, assume that there are only two firms operating in the market of oligopolistic competition. In this case $P(Q) = a - b(q_1 + q_2)$: Equation (4.9) takes the form $q_1 = \frac{a - bq_1 - bq_2 - MC_i}{b}$. After some transformations we obtain the equation of the oligopoly optimum if there are two firms:

$$q_1 = \frac{a - MC_i}{2b} - \frac{q_2}{2}. \qquad (4.10)$$

A graphical representation of equation (4.10) is called the ***reaction function.*** It shows the company's maximizing sales volume at a given competitor's sales volume. Let's depict the reaction functions of both firms in coordinates $q_1 - q_2$. The first firm's reaction function will be RF_1, and the reaction function of the second firm will be RF_2. Graphically, these equations are presented in Fig.4.8.

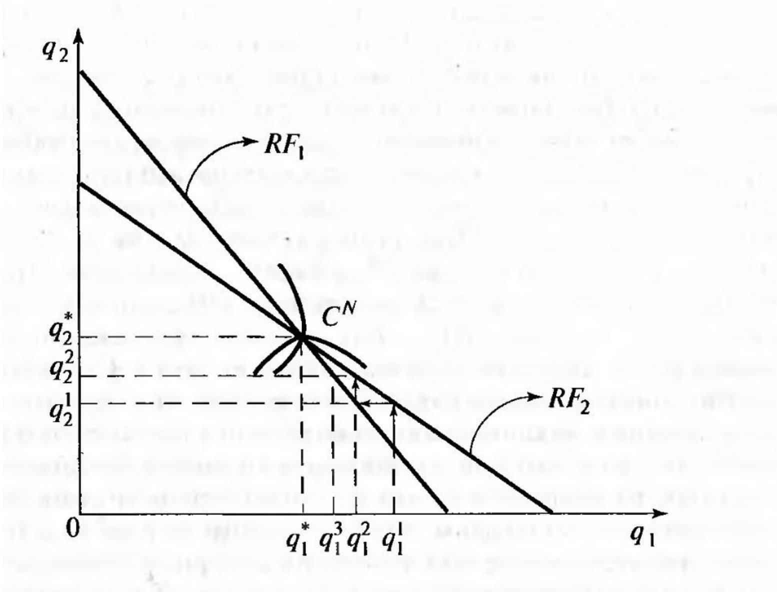


Fig.4.8. Oligopoly equilibrium in the case of two firms

As can be seen from Figure 4.8, the equilibrium is established at the point where the reaction functions of the two firms intersect. This type of market interaction is called ***Cournot equilibrium***. Being optimal for each firm individually, the Cournot equilibrium does not provide an optimum for the industry as a whole.

Example 4.7. There are two companies operating in the calculator market: Alfa and Vega. The demand for calculators is determined by the formula $P = 100 - 2(q_1 + q_2)$. The firms' cost functions have the forms $TC_1 = q_1^2$; $TC_2 = 2q_2^2$. Find the Cournot equilibrium.

Solution. Find the profit functions of both firms:

$\pi_1 = (100 - 2q_1 - 2q_2)q_1 - q_1^2 = 100q_1 - 2q_1q_2 - 3q_1^2.$

$\pi_2 = (100 - 2q_1 - 2q_2)q_2 - 2q_2^2 = 100q_2 - 2q_1q_2 - 4q_2^2.$

Next, we find the derivatives of profit functions by the firms' own sales volumes:

$\frac{\partial \pi_1}{\partial q_1} = 100 - 2q_2 - 6q_1.$

$\frac{\partial \pi_2}{\partial q_2} = 100 - 2q_1 - 8q_2.$

These functions illustrate the firms' reaction lines. Find the intersection of the reaction lines:$100 - 2q_2 - 6q_1 = 0;$

$q_1 = 16\frac{2}{3} - \frac{1}{3}q_2;$

$100 - 2q_1 - 8q_2 = 0;$

$q_1 = 4q_2 - 50.$

Equate two equations to each other: $4q_2 - 50 = 16\frac{2}{3} - \frac{1}{3}q_2.$

$4\frac{1}{3}q_2 = 66\frac{2}{3}; q_2 = 15{,}38.$

Let's express q_1

. $q_1 = 11{,}54.$

Hence the market price is: $P = 100 - 2 \cdot 26{,}92 = 46{,}16.$

Answer. The market will set a price of 46.16 rub. for one calculator with the sales volume 11,54unit of calculators Alpha and 15.38 units of calculators Vega.

According to the Cournot model the firms behave independently from each other. However, in practice, the oligopoly market often has a leader that all competitors follow. For example, in Russian practice, many smaller banks lower or raise their interest rates following PAO Sberbank. This type of leadership is taken into account in the Stackelberg model of price leadership proposed in 1934.

This model introduces the concept of an isoprophyte. An ***isoprofit*** is geometric location of points in coordinates $q_1 - q_2$, at which one of the firms makes the same profit. Isoprofits for the case of two firms are shown in Fig. 4.9.

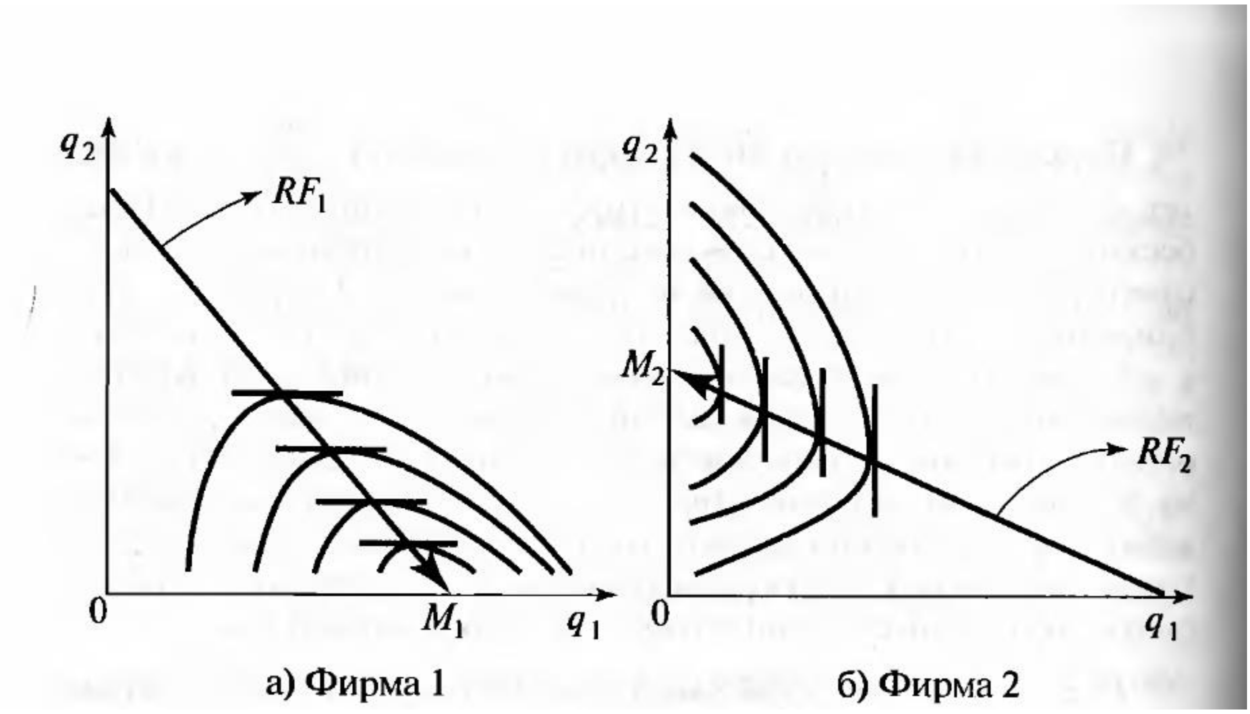


Fig.4.9. Graphic representation of isoprofits

Suppose that in the Stackelberg model firm 1 is the leader. In this case firm 2 chooses a point on the reaction line RF_2at which its profit is optimal. This point is the point of contact of the isoprofit of firm 2 and the reaction line of firm 1. A graphical representation of the Stackelberg equilibrium is given in Fig.4.10.

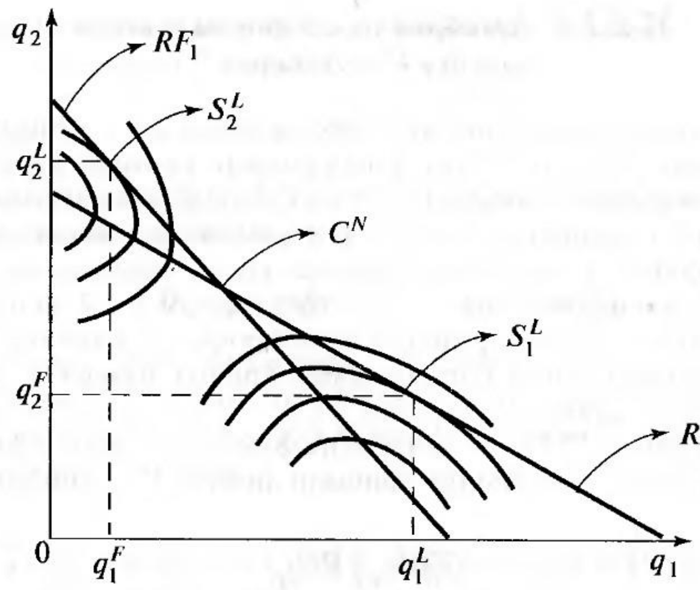


Fig.4.10. Graphic representation of the Stackelberg equilibrium

As can be seen from Fig.4.10, the Stackelberg equilibrium is reached at pointS_1^L. In this case, the leading firm will have a bigger profit than the following firm.

In the Cournot and Stackelberg models, the price is determined by the competition of firms for market share. At the same time, there is another type of models in which the sales volume of each seller is set as a result of the competition of firms on price. This model was first proposed by J. Bertrand in the late 19th century.

Suppose there are two manufacturers on the market which compete with each other on price. q_1 is the sales volume of the first manufacturer, P_1 is the price this manufacturer set for its products. Accordingly, q_2 is the sales volume of the second manufacturer, P_2 is the price of its products. Let's graphically represent the demand function for the first manufacturer's products (Fig.4.11).

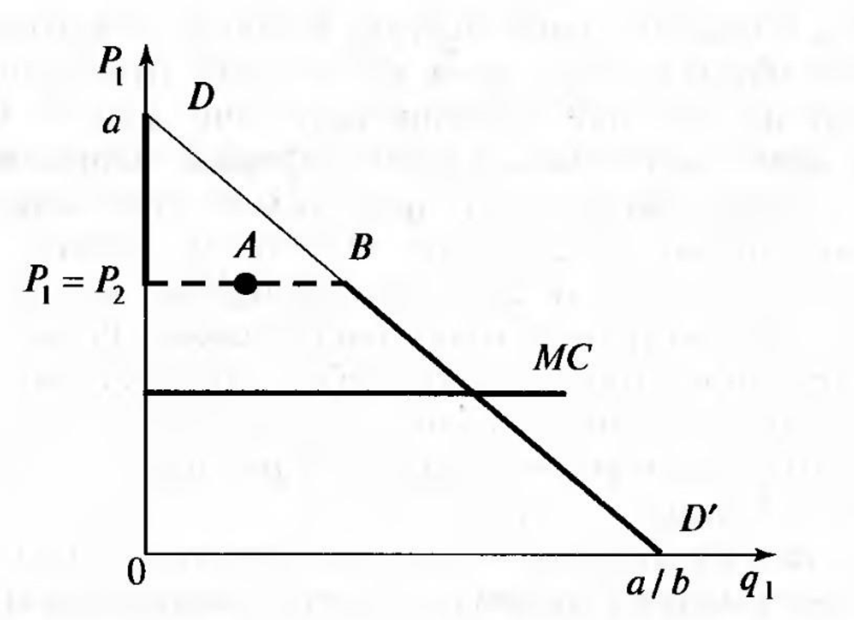


Fig.4.11. Demand for the products of a particular company according to the Bertrand model

Both firms operate on the market of homogeneous products, so if the first manufacturer sets a much higher price than the second one, then the demand for its products will be zero. In this case, all potential buyers will purchase goods from the second seller. If the first firm sets a significantly lower price than its competitor, the demand for its products will be equal to the demand for the products of the entire industry.

The Bertrand equilibrium is reached at the point where the price of the first seller is equal to the price of the second seller. If they use relatively similar production technologies, they will divide the market equally.

Based on the Bertrand model, the demand function for the products of the first firm can be represented as

$$d(P_1, P_2) = \begin{cases} 0; \text{если } P_1 > P_2. \\ \dfrac{D(P_1)}{2}; \text{если } P_1 = P_2. \\ D(P_1); \text{если } P_1 < P_2. \end{cases} \qquad (4.11)$$

So we have considered oligopoly models in which firms operate independently of each other. However, in practice, firms may enter into a cartel agreement. Meanwhile, firms agree on the price for products from all manufacturers and divide market shares among themselves. The purpose of cartel agreement between firms is to maximize the total profit.

Let MC_ibe the marginal costs of the *i*-th participant in a cartel agreement. Hence, $MC = \sum_{i=1}^{n} MC_i$ is the total marginal costs of the entire cartel. A general management body of the entire cartel distributes the sales volume among its participants so that at this point the marginal profit equals marginal costs: $MR = MC$. At this point, the profit of the entire cartel is maximized. However, the issue of profit distribution among cartel participants is the subject of an agreement between them.

Example 4.8. There are 10 manufacturers in the shoe market with similar cost functions: $MC_i = 10 + q_i$. The demand function in this market is: $Q = 1000 - 20P$. Suppose the firms decide to merge into a cartel. Find the sales volume of the entire cartel.

Solution. The general function of the marginal costs of the cartel has the form $MC = 100 + Q$. Cartel revenue $R = (20 - 0{,}05Q)Q = 20Q - 0{,}05Q^2$. The marginal profit of the cartel $MR = 20 - 0{,}1Q$.

The cartel's profit is at its maximum at the point where $MR = MC$.

$100 + Q = 20 - 0{,}1Q$;

$1{,}1Q = 120$;

$Q = 109$.

Answer. The sales volume of the entire cartel will be 109 pairs of shoes.

We have considered the main types of market structures. In the next chapter, we investigate special models of microeconomics: the nature of profit and choice under uncertainty.

TASKS FOR SELF-SOLVING FOR CHAPTER 4

1. Sauerkraut is sold in the market of perfect competition. Demand function for sauerkraut is as follows$Q = 1000 - 5P$. The supply of sauerkraut is described by the formula $Q = -80 + 4P$. Calculate the surplus of the consumer in this market.

2. AO Morozko is a monopolist in branded ice cream market. The company's cost function is: $TC = 12{,}5 + 0{,}5Q^2$. The function of the demand for ice cream is as follows: $Q = 10 - P$. Find the optimum of the monopolist and the profit made.

3. The demand of foreign tourists for the services of the Tsentralnaya Hotel is described by the formula$P = 16 - 2Q$. The demand of Russian tourists*:*$P = 10 - Q$. The hotel's marginal costs are constant and amount to 4 monetary units. Find the prices and sales volume of the Tsentralnaya Hotel for domestic and foreign tourists.

4. The demand for an attraction in an amusement park is $Q = 24 - P$for adults and $Q = 24 - 2P$for children. The marginal costs for the attraction service are constant and amount to 4 monetary units. Find the prices for the attraction for adults and for children.

5. There are two firms operating in the champignon market: firm *A* and firm *B*. The market demand for champignons is represented by equation $P = 100 - (q_1 + q_2)$. The cost function of both firms is the same: $TC_i = 4q_i$. Calculate the total sales volume in the market and the profit of each firm.

6. Suppose there are firm *A* and firm *B operating on the market.* The market demand in the market is represented by equation*:*$P = 100 - (q_1 + q_2)$. The cost

function of the first firm is $TC_1 = 20q_1$.The cost function of the second firm is $TC_2 = 20q_2$. Calculate the total sales volume in the market and the profit of each firm.

SELF-CHECK TEST FOR CHAPTER 4

1. If the apple market is competitive, in order to maximize profits, an apple producer has to set the price at the level of:

a) marginal utility of consumers;
b) marginal revenue;
c) marginal costs;
d) average costs.

2. Which of the following properties inevitably present in the market of perfect competition?

a) a large number of sellers;
b) a large number of buyers;
c) homogeneity of the product;
d) the products of different manufacturers have different prices.

3. In case of an increase in the excise tax on excisable products:

a) the price increases and the sales volume decreases;
b) the price and sales volume grow;
c) the price and sales volume fall;
d) the price increases and the sales volume decreases.

4. In which case does the firm appropriate the entire consumer surplus?

a) in a market of perfect competition;
b) in a monopoly market with the first-degree price discrimination;
c) in a monopoly market with second-degree price discrimination;
d) in a monopoly market with the third-degree price discrimination.

5. As the sales volume increases, the marginal revenue of the monopoly:

a) decreases;
b) increases;
c) can either increase or decrease;
d) not enough information to answer.

6. What limits the price a monopolist can set?

a) by consumer demand;
b) by the monopolist's average costs;
c) by the monopolist's average fixed costs;
d) by the monopolist's average variable costs.

7. Which of the following markets is likely to be an oligopoly?

a) of aircrafts;
b) of oil;
c) of cars;
d) of educational services.

8. Compared to the market of perfect competition, the monopolized market is characterized by:

a) higher prices and higher sales volume;
b) higher prices and lower sales volume;
c) lower prices and lower sales volume;
d) lower prices and higher sales volume;

9. The price elasticity of demand at the monopolist's price is -2. Marginal costs at the current output are 1 rub. per unit. In this case the profit-maximizing monopolist will set the price of:

a) 2 rub.;
b) 1 rub.;
c) 0,5 rub.;
d) 1,5 rub.

10. A profit-maximizing monopoly sets the same price as in the market of perfect competition if it uses:

a) first-degree price discrimination;
b) second-degree price discrimination;
c) third-degree price discrimination;
d) not enough information to answer.

11. Suppose cinema Formula Kino sells discounted tickets for people aged 16-22. Other visitors can only buy tickets at a standard price. In this case, the cinema uses:

a) first-degree price discrimination;
b) second-degree price discrimination;
c) third-degree price discrimination;
d) not enough information to answer.

12. As part of a promotional event, alcohol store Lion sets a 10% discount to all customers who purchase products in this store for more than 1,000 rubles per month, and 20% to those who purchase products for more than 2,000 rubles per month. In this case, the alcohol store uses:

a) first-degree price discrimination;
b) second-degree price discrimination;
c) third-degree price discrimination;
d) not enough information to answer.

13. The demand for CDs is represented by equation: $Q = 10 - P$. There are two competing manufacturers in an industry as specified in the Cournot model. The marginal costs of both manufacturers are 3 rubles. In this case the market price will be:

a) 7 rub.,
b) 5 rub.,
c) 14 rub.,
d) 6 rub.

14. A monopolist maximizes profit. Elasticity of demand by price at the optimum point is -3. In this case, the price set by the monopolist will be higher than its marginal costs by:

a) 10 %,
b) 50 %,
c) 30 %,
d) 100 %.

CHAPTER 5. MODELING FIRM PROFITS AND FACTOR MARKETS

There are three main factors due to which a firm makes high profits: monopoly power, favorable market conditions, and technological superiority over competitors. Let's take a closer look at the key concepts and models of the three components of the company's profit.

The issue of dividing revenues into monopoly profit, boom profit and technological profit has not only theoretical, but also practical significance. Shareholders and potential investors should know what causes the growth or decline in the company's profits.

5.1 Definition of monopoly profit

According to the first approach, monopoly profit is the difference between the profit made by a monopolist and the profit that this firm would make without being a monopoly. The disadvantage of this approach is that it is almost impossible to calculate the profit that an existing monopolist would receive if operating in a highly competitive environment (that is, not being a monopolist).

The second approach to determining monopoly profit is calculating the difference between the actual profit and the profit that the firm would have made from selling the same total of output but at a competitive market price. However, theoretically, this approach has its drawbacks because the sales volume of a monopolist is usually lower than the potential sales of the same firm should it operate in a market of perfect competition.

According to the third, the most popular approach, the monopoly profit is equal to the current sales volume multiplied by the difference between the price and marginal costs:

$$\pi_M = Q(P - MC), \tag{5.1}$$

where π_M is the monopoly profit.

As the monopolist sets the price above marginal costs, the profit per unit of products sold is equal to the difference between the price and marginal costs. The calculation of the Lerner index follows the same logics.

Meanwhile, the company's entire profit may consist of the sum of two components: monopoly profit and non-monopoly profit:

$$\pi = \pi_M + \pi_N, \tag{5.2}$$

where π is the total company profit, π_N is the non–monopoly company profit associated with favorable market conditions.

The total profit of the company is equal to the difference between the price and average costs multiplied by the sales volume: $\pi = (P - AC)Q$. With the help of formula (5.1), it is possible to derive the equation for non-monopoly profit:

$$\pi_N = Q(MC - AC). \tag{5.3}$$

Practical application of formulas (5.1) and (5.3) is difficult because a manufacturer does not know the cost function, and therefore its marginal costs either. This difficulty can be overcome by several special situations that would help to approximately estimate the amount of monopoly and non-monopoly profits.

1) Consider the case when a company's sales volume changes over several months, but its average costs remain unchanged. In this case we can assume that there is a constant returns on scale. Consequently, the firm is at the point of minimum average costs equal to the marginal costs. Thus, the entire profit is a non-monopoly one.
2) The dynamics of average costs act in the opposite direction to the sales volume. It evidences that the firm operates under positive returns on scale. In this case, the entire profit is monopolistic.
3) To estimate the cost function we can apply the least squares method. Notably, to make the assumption that other factors are constant work we have to consider a relatively short period of time.

Nevertheless, the amount of marginal costs is not included in the company's financial statements.

Example 5.1. The cost function of the firm is described by equation $TC = \frac{1}{2}Q^2 - 4Q + 10$. The demand function for the company's products is as follows$P = 10 - Q$. The company sets the price of 6 rubles. per unit of production. Find the company's monopoly profit.

Solution. To find the monopoly profit, it is necessary to calculate the profit that a firm would have made under perfect competition. For this, it is necessary to calculate the sales volume and the price at which $MR = MC$.

$MC = \frac{dTC}{dQ} = Q - 4.$

$R = QP = 10Q - Q^2.$

$MR = 10 - 2Q.$

$4 - Q = 10 - 2Q;$

$Q = 6;$

$P = 4.$

$\pi = 6 \cdot 4 - 18 + 24 - 10 = 20.$

Now let's calculate the profit that a monopolist would have made under perfect competition: $P = MC$; $6 = Q - 4$; $Q = 10$; $P = 0$. Thus, the entire profit is a non-monopoly one.

Answer. The firm makes a monopoly profit of 20 monetary units.

5.2 Definition of technological profit

Let's now consider a situation where a firm makes a profit due to the introduction of new technologies and technological advantages over competitors. In this case, the company's average costs per unit of production are lower than those of its competitors. Besides, the firm's average costs can be lower because it has access to cheaper resources than its competitors.

So the profit in this case is a monopoly profit as the firm has a monopoly access to cheap resources or unique technologies.

Example 5.2. The largest credit institution in Russia is PAO Sberbank. In the period between 2016 and 2019, its profits were from 516 billion to 870 billion rubles. The profitability of its assets is higher than that of most private banks. This is largely determined by the fact that Sberbank has the opportunity to attract deposits at a lower interest rate because its deposit liabilities are guaranteed by the state and public confidence in this financial institution is higher than in private banks.

Let's differentiate between two types of monopoly profits: the profit associated with market dominance and the profits associated with access to exceptional technologies. Here D is the demand for the company's products, MC is its marginal costs. AC is the company's average costs and $\overline{AC}$ is he average costs of its competitors. The graphical differentiation of profit is shown in Figure 5.1.

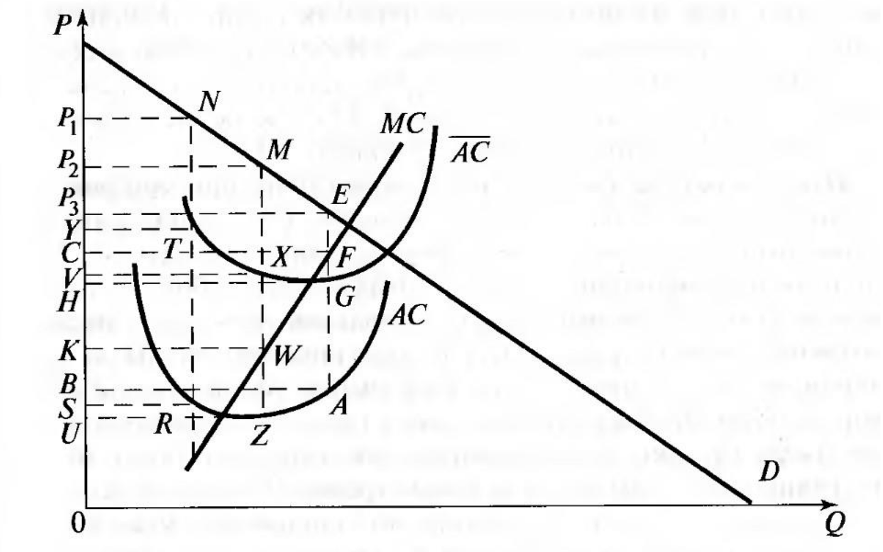


Fig.5.1. Definition of monopoly profit

Suppose, the firm sets price P_3. This price is higher than MC, AC и $\overline{AC}$. As can be seen from Fig.5.1, the company's curve of average costs lies lower than the curve of average costs of its competitors. The manufacturer is definitely to receive a part of its profit due to technological advantages or monopoly access to cheap resources.

Thus, value Q$(P - \overline{AC})$can be defined as the profit associated with market dominance, and value Q$(\overline{AC} - AC)$ as the profit associated with a technological advantage.

Next, there may be a problem related to the distribution of profits received through a unique technology and profits received through access to cheap resources. To solve this problem, we should calculate the company's costs, provided that it will purchase raw materials at the same prices as its competitors. Thus, the effects of the price difference are eliminated. However, carrying out such calculations in practice is extremely difficult.

Example 5.3. A company has a unique technology and sets the price for its products at 8 monetary units. The average costs of the company are 7 monetary units. The average average costs of its competitors are 7.5 monetary units. The output volume is 100 units. Find the company's profit received due to its technological advantage.

Solution. In this case $\overline{AC} = 7{,}5$;

$AC = 7$;

$P = 8$;

$Q = 100$.

Hence $\pi = 100 * (7{,}5 - 7) = 50$.

Answer. Due to the unique technology, the company makes a profit of 50 monetary units.

5.3 Demand for factors of production

This section analyzes the pricing process for the main factors of production – labor and capital. In commodity markets, companies are sellers, and households are buyers. Labor market is different. Here firms, are buyers and particular individuals act as sellers.

Suppose the revenue and costs of a firm is determined by the amount of labor and capital it uses. Mathematically, this can be written as follows: $\pi = R(K, L) - TC(K, L)$. Then, in order to find the volumes of factors of production that maximize profit, we should equate the following partial derivatives to zero:

$$\begin{cases} \dfrac{\partial \pi}{\partial L} = \dfrac{\partial R}{\partial L} - \dfrac{\partial TC}{\partial L} = 0. \\ \dfrac{\partial \pi}{\partial K} = \dfrac{\partial R}{\partial K} - \dfrac{\partial TC}{\partial K} = 0. \end{cases} \qquad (5.4)$$

Thus, the firm has to increase the amount of labor and capital used until the additional income from the use of an additional unit of each of the factors of production equals the costs of its purchase.

Next, since $R = R(Q)$ и $Q = Q(K, L)$, then by the rule of differentiation of a complex function

$$\frac{\partial R}{\partial L} = \frac{\partial R}{Q} \cdot \frac{\partial Q}{\partial L} = MR \cdot MP_L. \qquad (5.5)$$

Value $MR \cdot MP_L$ is called the marginal profitability of labor. It shows by how many monetary units the company's revenue will increase when one additional employee is hired.

The company's marginal costs from hiring one additional employee are equal to the pay rate: $\frac{\partial TC}{\partial L} = w$. At the same time, we assume that all employees of this company have the same wages. Mathematically, to find the amount of labor that maximizes profit we should solve the equation

$$MR \cdot MP_L = w. \tag{5.6}$$

Market wage of a particular employee consists of two components: their labor productivity and demand for the products of the company, where they work. Thus, workers of in-demand jobs may earn wages that are too high in relation to their labor productivity. Accordingly, employees of specialties that are in low demand will receive lower wages.

Let's assume that the pay rate in the labor market has increased. Consequently, the demand for labor has decreased. This can be caused by two effects: the substitution effect and the output effect. The substitution effect is due to the fact that with an increase in the cost of labor, it becomes more profitable for a company to replace labor with capital. The output effect is related to the fact that an increase in the average market wage leads to an increase in costs and, consequently, the firm reduces its production activities.

In the vast majority of cases, the demand for labor negatively depends on the price of this factor of production: $\frac{\partial L}{\partial w} < 0$. At the same time, the cross effect has a different sign. The demand for labor positively depends on the interest rate on borrowed capital: $\frac{\partial L}{\partial r} > 0$. This is due to the fact that with the rise in the cost of capital, the firm is forced to reduce its use, replacing it with labor.

Let's now imagine the demand curve for the firm's labor in coordinates *w–L*. As mentioned above, the demand for labor is determined by equation (5.6). The expression $MR \cdot MP_L$ is called marginal productivity of labor and is indicated by MRP_L. The graph MRP_L is shown in Fig.5.2.

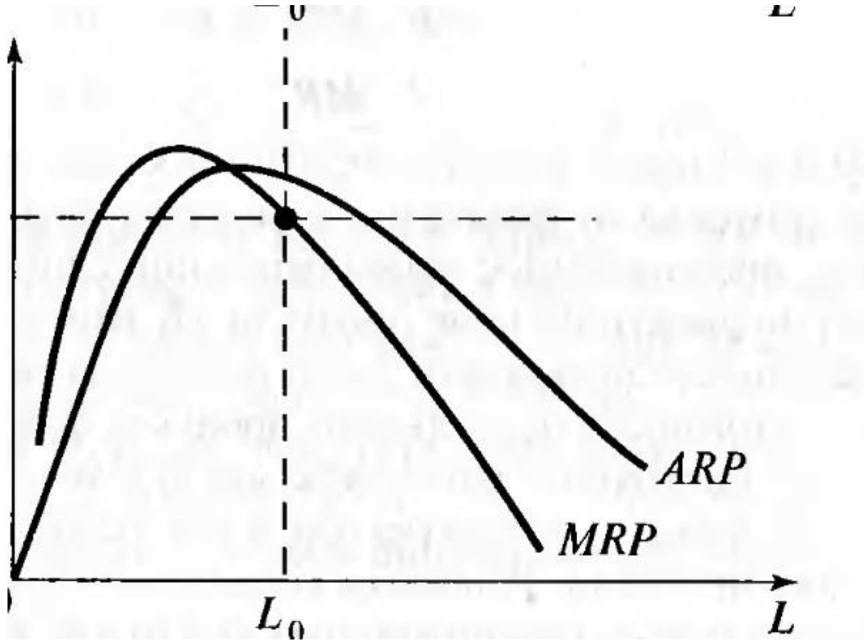


Figure 5.2. Graphic representation of the marginal productivity of the production factor

The labor demand curve coincides with the curve MRPshown in Figure 5.2. Suppose that a hiring labour firm sells its products in a competitive market. Then the condition (5.6) turns into an expression

$$MP_L = \frac{w}{P}. \tag{5.7}$$

Thus, the demand for labor depends on the following factors: capital accumulation, inflation in the country, and scientific and technological progress. Let's look at the impact of these determinants in more detail.

With an increase in the capital used, the demand for labor falls. For example, if peasant labor is replaced by mechanized labor of tractor drivers, the demand for workers in agriculture falls.

The demand for labor positively depends on the inflation rate in the country. With an increase in the price a firm has to hire more workers. It should be noted that this rule does not apply in hyperinflationary economies.

With the transition to a new technological level, the need for labor decreases. For example, automation in the field of accounting has led to the decrease in the demand for qualified accountants.

Example 5.4. Suppose that blueberries are a product of a competitive industry. The production function of blueberry harvesting is $y = 2\sqrt{L}$. The market price of blueberries is 60 monetary units. Derive a labor demand equation for a blueberry-picking company.

Solution. As it was shown above, for a competitive commodity market, labor demand is found by formula $MP_L = \frac{w}{P}$. Let's calculate the marginal product of labor: $MP_L = \frac{1}{\sqrt{L}}$.

Hence, $\frac{1}{\sqrt{L}} = \frac{w}{60}$;

$\sqrt{L} = \frac{60}{w}$;

$L = \frac{3600}{w^2}$.

Answer. The labor demand function is as follows $L = \frac{3600}{w^2}$.

The question arises: what is each of the factor's contribution to the output? The answer to this question is given by Euler's theorem, according to which the following equality must be satisfied

$$MP_L \cdot L + MP_K \cdot K = Q. \quad (5.8)$$

Thus, the contribution of each factor to the production technology is determined by the type of production function.

5.4 Supply of factors of production

According to the modern economic theory, from an individual's viewpoint, labor is an anti-good. The more a person works, the less utility they obtain. Someone may disagree with this claim, arguing that the labor process can sometimes be enjoyable. But then the question arises: why don't we work days and nights?

The majority of people need jobs primarily in order to have the money to buy goods and services. However, if a person has to increase the amount of time spent on work, they sacrifice rest and leisure. Meanwhile, it is assumed that there are plenty of different job offers on the labor market providing different number of working hours per week.

Let's represent the described problem mathematically. Suppose L^e is the number of hours per week that an individual spends on rest and leisure. Then L is the average number of hours per week that an individual spends on work. T is the total number of hours per week. Then the following equality is satisfied

$$L + L^e = T. \quad (5.9)$$

We assume that all hours worked are paid to the individual at pay rate *w*. It should be noted that the real situation in the domestic labor market may look different. Typically, employers offer their employees the maximum possible number of hours per week provided for by Russian legislation. Overtime work is not supposed to be paid.

Variable *C* is an average cost of goods and services that a consumer consumes during a week. It becomes obvious that an employee has to choose between rest and consumption of goods and services. In order to consume more, a person is forced to work more. However, the person loses hours of rest.

In this case, budgetary constraint of an employee is

$$C + wL^e = wT. \qquad (5.10)$$

It follows from this formula that the pay rate is the price of the rest and leisure good. It must be noted that our assumption here is that all employers evaluate work skills and abilities of an employee equally. However, in practice it happens that different employers may evaluate different labor abilities of an employee differently.

The inclination angle of this budget line is determined by the pay rate. Nevertheless, unlike the budgetary constraint line for the selection of commodity sets, the budget set boundary is limited to the maximum possible number of hours *T*.

Suppose that there is an individual's utility function of two economic goods – consumption and leisure. An employee seeks to maximize it under a time limit:

$$\begin{gathered} U(L^e, C) \to max \\ C + wL^e = T. \end{gathered} \qquad (5.11)$$

So the graphic representation of the employee's choice of the optimal amount of working time is shown in Fig.5.3.

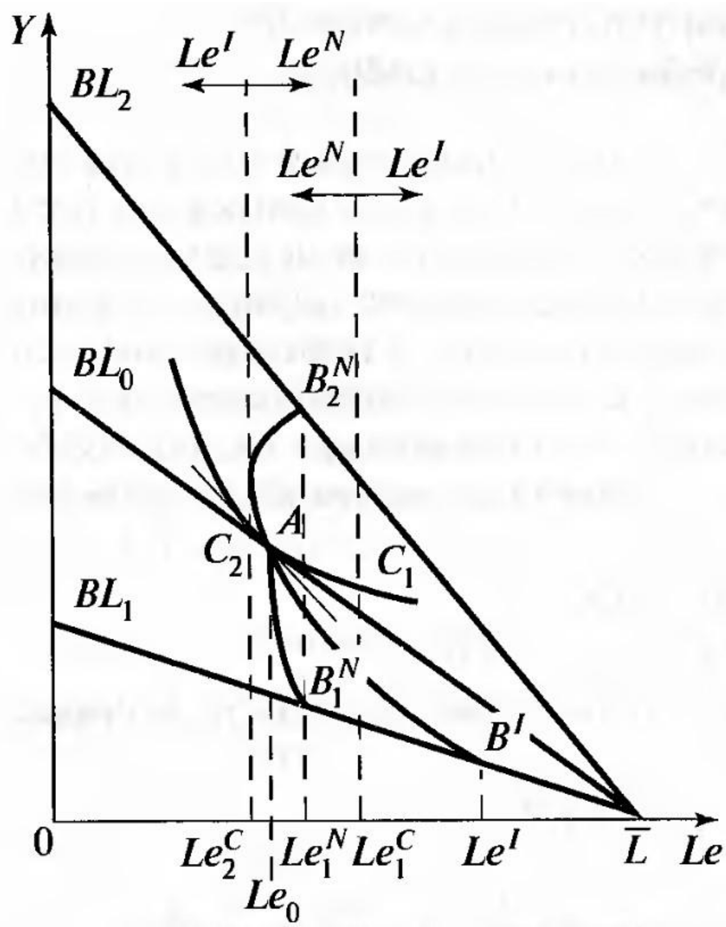


Fig.5.3. Graphic representation of the optimal working time choice

Look at Fig.5.3. Suppose that initially, at the pay rate, the individual's optimum was at point A. With this choice, he/she spends L_0^ehours on leisure. Suppose the pay rate has decreased. Then the budget constraint line shifts down but at the same time it starts from point T. In this case, the consumer will choose less cheap leisure and reduces consumption due to the fact that his income has decreased.

Now suppose that the pay rate has increased. Consequently, the individual's budget constraint line shifts upwards. In this case, the amount of leisure also decreases while the consumption increases. This is due to the fact that with different income rates, the demand for recreation has different responses to increases in individual well-being.

For example, in the USA and Western Europe in the 20th century, the average working week was getting longer and by the mid-century an employee worked 55-60 hours per week. However, even with such a workload villagers migrated to cities.

After the Second World War, working week in most countries of Western Europe and the United States was reduced to 40 hours. This first led to the fact that many employees were looking for various kinds of side jobs to increase their income. However, by the end of the 20th century. this trend finished. At the beginning of the 21st century in some countries of Western Europe, the maximum working week at one workplace decreased to 36 hours.

Clearly, during the second half of the 19th and in the 20th centuries, the average real income in most countries of Western Europe increased. Thus, with a low income, leisure is a low-value commodity. However, as an individual's wealth increases, leisure turns into a normal good.

The labor supply graph has the form of a curve. This fact is shown in Figure 5.4.

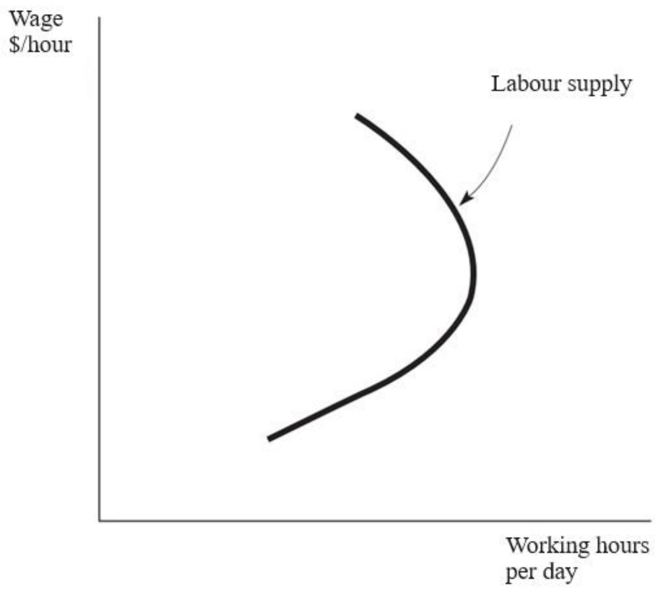


Fig.5.4. Labor supply curve

As can be seen from Fig. 5.4, as salary increases, the labor supply curve bends.

Example 5.5. Mr. Ivanov's utility function is as follows $U = \sqrt{CL^e}$. Employers evaluate Ivanov's labor with a salary of 5 rubles. Mr. Ivanov can work as much as 14 hours a day. How many hours a day will Ivanov work on average?

Solution. Let's find the individual's marginal utilities on leisure and consumption:

$MU_C = \frac{\sqrt{L^e}}{2\sqrt{C}}$;

$MU_{L^e} = \frac{\sqrt{C}}{2\sqrt{L^e}}$.

As is known from the previous chapters, optimum is achieved at the point where the ratio of marginal utilities is equal to the ratio of prices:

$\frac{MU_C}{MU_{L^e}} = 5;$

$$\frac{2\sqrt{C * C}}{2\sqrt{L^e L^e}} = 5;$$

$\frac{C}{L^e} = 5; C = 5L^e.$

Next, we obtain the budget constraint line:

$C + 5L^e = 5 * 14 = 70.$

Hence, $10L^e = 70;$

$L^e = 7;$

$L = 14 - 7 = 7.$

Answer. On average Ivanov will work 7 hours a day.

Labor supply in any professional field is determined by adding up labor supply curves of individual individuals.

5.5 Equilibrium in factor markets

In general, the analysis of market equilibrium in the labor market is similar to the corresponding analysis of commodity markets. However, it must be noted that the labor supply curve is bent. In this case, several market equilibria are possible.

Let's consider a situation when the labor supply curve intersects the demand curve in two points (Fig.5.5).

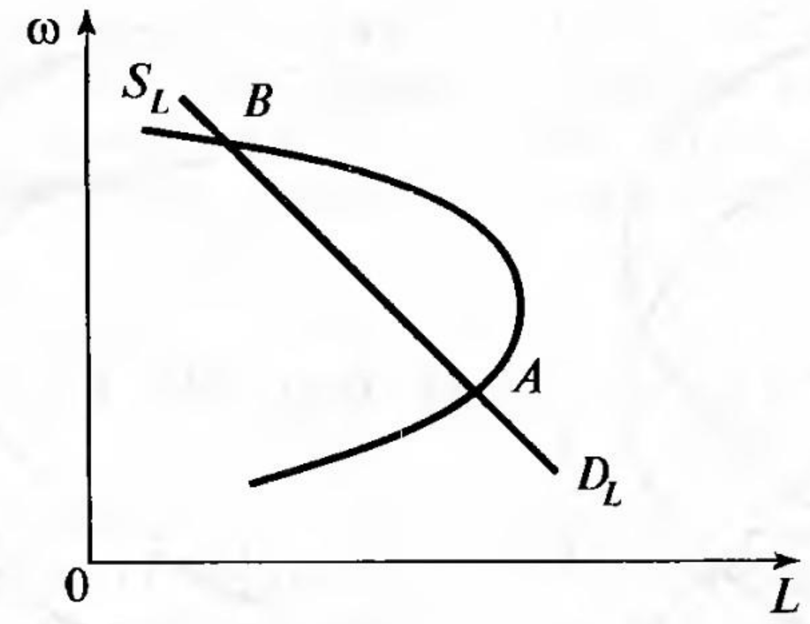


Fig.5.5. Equilibrium in a competitive labor market

As Fig.5.5 shows, equilibrium in a competitive labor market can be achieved at two points: A and B. Suppose the equilibrium is reached at point B. However, temporarily the pay rate turned out to be higher than the equilibrium one. In this case, demand exceeds supply. In this situation, according to the laws of the equilibrium market, wages should continue to grow. Therefore, equilibrium at point B is not stable.

If the balance of supply and demand is established at point *A*, under the influence of market mechanisms this equilibrium is stable. Suppose that the salary turned out to be higher than the one that is set at point *A*. In this case labor supply exceeds the demand. This, in turn, leads to lower wages and the market returns to point *A*.

By analogy with the consumer surplus in the commodity market, in the labor market there is a concept of rent. ***Rent*** is a benefit received by the owner of a factor of production from its sale on the market. In terms of labor market, rent is the benefit received by employees from different market mechanisms used for their hiring.

Suppose Mr. Petrov is ready to work as a hired accountant for a salary of 30 000 rubles per month. However, due to the competitive labor market, the average salary of an accountant in his city is 40 000 rubles. In case there are no wage-setting mechanisms, Petrov will work for 30 thousand rubles. However, due to the competition of employers for employees, his salary will be 40 000 rubles. In this case, Petrov's rent is 10 000 rubles. A graphical representation of rent in the labor market is shown in Fig. 5.6.

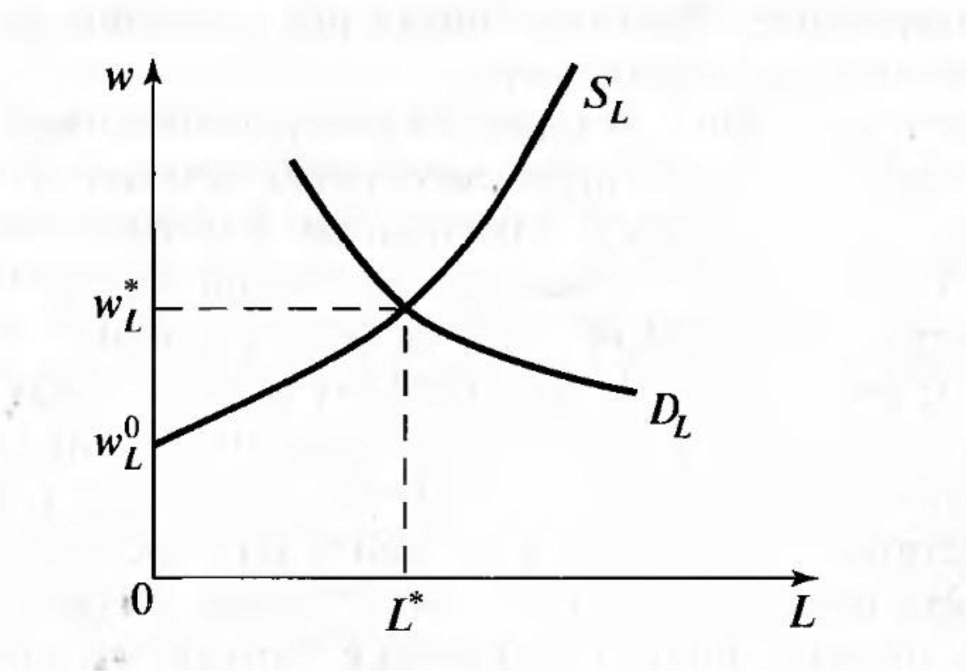


Figure 5.6. Rent in a competitive labor market

The rent in the labor market shown in Figure 5.6 is equal to the area of the figure that lies between the pointsw_L^*, w_L^0, L^*.

Meanwhile, it should be kept in mind that a monopsony model may lack rent. If employees can sell their labor to only one employer, then the employer can pay them the minimum wage for which they agree to work. In Russia, such a situation is quite typical in so-called single-industry towns.

Example 5.6. The short-term production function of a monopolist firm has the form$Q = 2\sqrt{L}$. The demand for company's products is given by the equation $P = 90 - Q$. The market pay rate is 9000 rubles. Find the optimal number of employees the monopolist firm will hire.

Solution. The labor demand function is determined by the ratio

$w = MP_L \cdot MR_x$.

The marginal product of labor is equal to: $MP_L = \frac{\partial Q}{\partial L} = \frac{1}{\sqrt{L}}$.

Let's find the company's revenue function:$R = P \cdot Q = 90Q - Q^2$;

$MR_x = 90 - 2Q$.

$MP_L \cdot MR_x = \frac{90-2Q}{\sqrt{L}} = \frac{90-4\sqrt{L}}{\sqrt{L}} = w = 9000.$

$9000\sqrt{L} = 90 - 4\sqrt{L}$;

$L = 0{,}01.$

Answer: the monopolist firm will hire 10 employees.

We have studied the mechanism of functioning of the labor market as the most important types of factor markets. The next chapter considers the theory of general economic equilibrium.

TASKS FOR SELF-SOLVING FOR CHAPTER 5

1. The demand for company's products is given by function $P = 10 - \frac{5}{3}Q$. The company 's costs are determined by the function $TC = 0{,}5Q^3 - 4Q^2 + 10Q$. Find total profit, monopoly profit and non-monopoly profit.
2. The labor demand curve is given by the ratio $L = -50w$+450. The labor supply curve is determined by the formula $L = 100w$. Find market values of wages and the number of employees.
3. Suppose, the short-term production function of a monopolist firm has the form$Q = 4\sqrt{L}$. The demand for this monopolist firm's products is as follows$P = 90 - Q$. Derive the labor demand function for this monopolist firm.
4. Mr. Petrov's utility function is as follows $U = C\sqrt{L^e}$. Petrov can spend on work maximum 15 hours. Employers can pay Mr. Petrov 20 rubles per hour for his work. How many hours per day will Petrov work?
5. Production function of a monopolist firm has the form$Q = \sqrt{KL}$. The demand function for the products of this monopolist firm is $Q = 100 - P$.. Market pay rate is 4 rubles per hour, the capital rental rate is 1 ruble. Find the optimal amount of labor and capital used.

SELF-CHECK TEST FOR CHAPTER 5

1. Suppose there was a technological boom in the economy of country *A*, which led to the emergence of a new production requiring labor costs. All other parameters of country *A* remained unchanged. In this case, in this country*:*
   a) labor demand increases;
   b) labor demand decreases;
   c) labor supply increases;
   d) labor supply decreases.
2. The state sets a minimum wage that exceeds the market pay rate in the country. In this case, in this country:
   a) unemployment rate increases;
   b) unemployment rate decreases;
   c) inflation rate accelerates;
   d) inflation rate slows down.
3. The optimum of a monopolist firm in the finished product market is achieved at the point where equality is fulfilled:
   a) $MP_L \cdot MR_x = w$;
   b) $MP_L \cdot P_x = w$;
   c) $\frac{MP_L}{MR_x} = w$;

d) $\frac{MP_L}{P_x} = w$.

4. The price that the firm sets exceeds its marginal costs. At the same time, the average costs of the firm exceed its marginal costs. In this case, the firm:
   a) makes a positive profit. It includes both monopoly and non-monopoly profit;
   b) makes a positive profit. It includes only monopoly profit;
   c) makes a positive profit. It includes only non-monopoly profit;
   d) makes a negative profit.
5. A firm makes both monopoly and non-monopoly profits. However, as a result of some event, the price of the firm's products decreased but marginal costs increased. In this case, the firm's profit:
   a) increased due to the increase in monopoly profits;
   b) increased due to an increase in non-monopoly profits;
   c) increased due to an increase in both monopoly and non-monopoly profits;
   d) decreased;
6. Labor demand of a firm does not depend on:
   a) the dynamics of demand for products that the firm produces;
   b) average market wages;
   c) manning of the company;
   d) the average time that employees of the company spend on rest.
7. Economic rent is equal to:
   a) the minimum cost of the factor that the company can pay to purchase it;
   b) the maximum cost of the factor that the company can pay to purchase it;
   c) the difference between the maximum and minimum costs of the factor;
   d) the difference between the actual cost of the factor and the minimum price that the firm can pay for it.
8. If a company is a monopsonist in the labor market, economic rent:
   a) is fully owned by this company,
   b) is fully owned by employees,
   c) it is divided equally between employees and the company,
   d) not enough information to answer.

CHAPTER 6. GENERAL EQUILIBRIUM, EFFICIENCY AND EXTERNALITIES IN MICROECONOMICS

In the previous chapters the concept of partial equilibrium in individual markets was considered. First, the equilibrium in the goods market was studied, followed by the investigation of factor markets. This chapter will consider how a change in supply or demand in one market will affect the equilibrium within another economic system.

If equilibrium values in the commodity market change, similar values for the factors of production will change too. For example, the decline in demand for crude oil in the first months of the COVID-19 pandemic led to a reduction in wages in oil companies. The rise in the price of natural gas results in the rise in prices for electricity and utilities.

The analysis of the general equilibrium provides answers to a number of questions. Firstly, is simultaneous equilibrium in all markets possible? If so, is it stable? Secondly, how effectively does the economy work in general equilibrium?

6.1 The existence of a general equilibrium

The relationship between different commodity markets is often mutually inverse. Suppose, butter and margarine are substitutes. In this case, when the price of butter rises, the price of margarine will rise too since the cross-elasticity between substitute goods is a positive value. However, an increase in the price of margarine will further increase the price of butter as well.

Depending on feedback actions, all inter-market interactions can be divided into symmetric and asymmetric. Symmetric feedbacks act in both directions in the same way. These, in turn, are divided into unidirectional and multidirectional.

Unidirectional feedbacks in related markets occur when an increase in the price of product *A* also leads to an increase in the price of product *B*. Such substitute products as sausage and ham can illustrate the point. An increase in the price of sausage leads to a simultaneous increase in the price of other meat gastronomy at the same time.

Multidirectional feedback occurs when an increase in the price of one product leads to a decrease in the price of another product. For example, an increase in prices for system units leads to a reduction in sales of personal printers and, as a result, a decrease in prices for them.

Asymmetric feedbacks are the type of inter-market interaction in which an increase in the price of product *A* leads to an increase in the price of product B, while an increase in the price of product *B* leads to a decrease in the cost of product *A*.

Suppose, economic good *A* is a public transportation service, and good *B* is an expensive French perfume. The increase in public transport fare does not lead to the fact that people will use it less. Consequently, consumer incomes will fall. This leads to a reduction in demand for expensive French perfume and, consequently, to a decrease in its price. At the same time, the rise in price of expensive perfumes leads to a sharp drop in demand for them and an increase in consumers' real incomes. This increases the demand for public transport services.

All types of interaction between different markets can be classified as one of the listed above types.

There is a concept of excess demand that helps to analyze inter-market interaction. Excess demand is represented by D_n. Excess demand is equal to the difference between supply and demand and can be defined as unsatisfied consumer desires in the market:

$$D_n = D - S. \tag{6.1}$$

Suppose, in the market of product *X* the price is set by the state. Thus, if the state has set the price above the equilibrium, an excess supply is created in the market. If the government has set a price lower than the one that would have been set as a result of market mechanisms, there is excessive demand. This situation is shown in Figure 6.1.

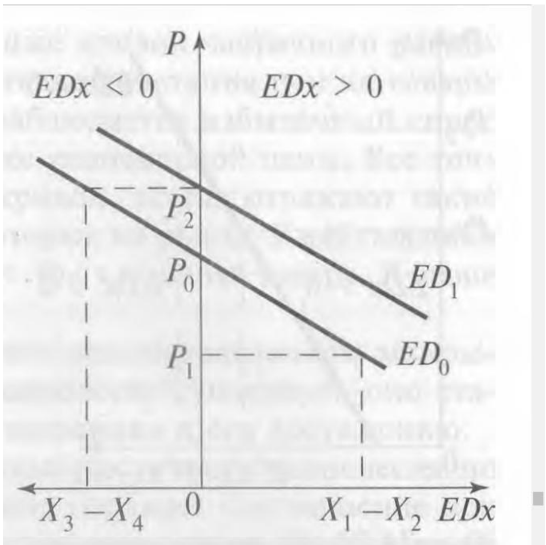


Fig.6.1. Excess demand depending on the price

In Figure 6.1, line ED_0 indicates excess demand. The equilibrium price is P_0. If it is so, the excess demand is zero. If the price of product *X* is less than P_0, excess demand is negative. If the price is lower than the market price, the demand exceeds supply.

Now let's assume that the price of the substitute product *Y* has increased. In this case, the demand for product *X* also increases. This leads to an increase in the market price up to P_2. In this situation, the excess demand line shifts up to ED_1. The following relationship can be identified: the higher the price of substitute product *Y*, the higher the price of product *X*, at which excess demand is zero.

Figure 6.2 shows the zero excess demand line in coordinates $P_X - P_Y$.

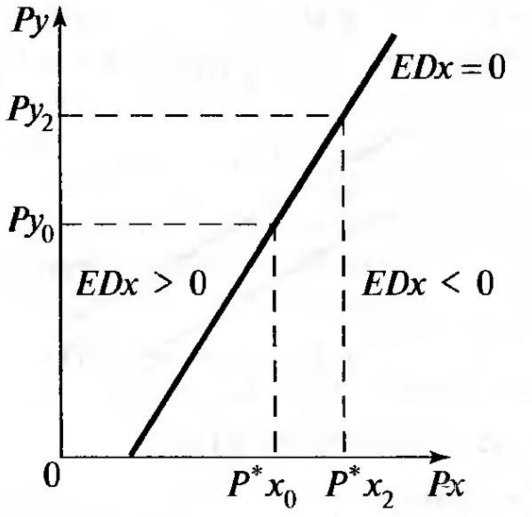


Fig.6.2. The combination of prices for goods X and Y at which excess demand is zero

Similar reasoning can be applied to product *Y*. In coordinates $P_X - P_Y$ another line can be drawn which would illustrate the conditions under which the excess demand for product *Y* is zero.

Let's fit together the lines of zero excess demand for goods *X* and *Y* in coordinates $P_X - P_Y$((Fig. 6.3).

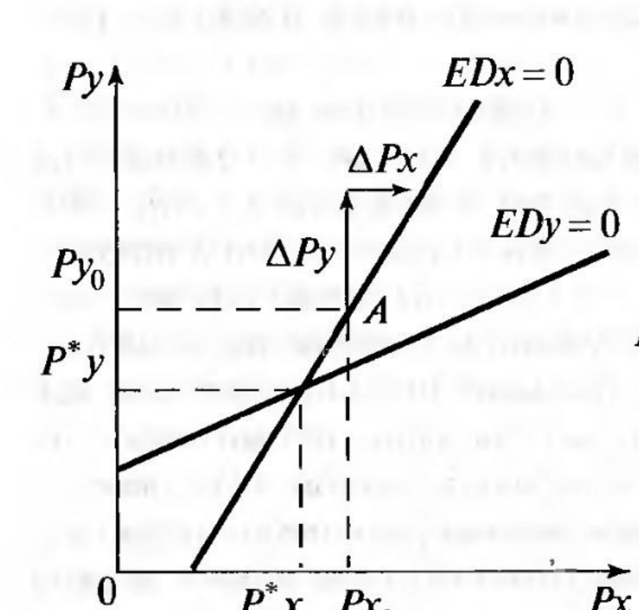


Fig.6.3. General equilibrium in the markets for products *X* and *Y*

The general equilibrium in the markets for products *X* and *Y* is reached at the intersection point of two lines of zero excess demand (Fig. 6.3).

Let's show that the equilibrium shown in Fig.6.3, is not stable. It should be noted that the general equilibrium is only possible if the inclination angle of line ED_x is greater than the inclination angle of lineED_y. Suppose, the price of product *X* has decreased. Since goods X and Y are substitutes, the demand for product *Y* has also increased. Consequently, its price has increased too. Thus, the actual ratio of prices for goods *X* and *Y* deviates further and further from the equilibrium point.

Example 6.1. Let's consider the interaction of markets of two products *X* and *Y*. The demand for product *X* is given by the equation $Q_X^D = 20 - P_X - 0{,}5P_Y$, the supply of this product is $Q_X^S = 10 + P_X$. Accordingly, the demand for product *Y* is given by formula $Q_Y^D = 30 - 0{,}5P_X - 0{,}25P_Y$, its supply is $Q_Y^S = 20 + 2P_Y$. At what prices for goods *X* and *Y* is the general equilibrium achieved in these commodity markets?

Solution. We derive the dependence of the equilibrium in the market for product *X* on parameters $P_X$and P_Y.

$20 - P_X - 0{,}5P_Y = 10 + P_X$;

$0{,}5P_Y = 10 - 2P_X$;

$P_Y = 20 - 4P_X$.

Next, we derive the dependence of the equilibrium in the market for product *Y* on parameters $P_X$and P_Y. $30 - 0{,}5P_X - 0{,}25P_Y = 20 + 2P_Y$;

$2{,}25P_Y = 10 - 0{,}5P_X$;

$P_Y = 4{,}33 - 0{,}2P_X$.

Let's find the intersection point of the two lines obtained. $20 - 4P_X = 4{,}33 - 0{,}2P_X$;

$3{,}8P_X = 15{,}67$;

$P_X = 4{,}12$;

$P_Y = 20 - 4 * 4{,}12 = 3{,}52$.

Answer. The general equilibrium is achieved if the price of product *X* is equal to 4.12 monetary units and the price of product *Y* is equal to 3.52 monetary units.

Let's imagine the general equilibrium model in a more abstract form. Suppose that there are *N* economic goods in some closed economy, some of which serve as resources for others. The supply and demand of each of the economic goods depend on the prices of other goods and services:

$$\begin{aligned} D_i &= f(P_1, P_2, \dots, P_n), \\ S_i &= f(P_1, P_2, \dots, P_n). \end{aligned} \tag{6.2}$$

In this case, equilibrium in all the markets is possible if there is vector $P = (P_1, P_2, \dots, P_n)$,for which the following equality is true

$$DN_i(P^*) = 0, i = 1,2, \dots, n. \tag{6.3}$$

Mathematically, in order to find vector P^*one should solve the system of equations, which is described by the formula (6.3). The resolvability of this system was proved by L. Walras.

6.2 Efficiency of general economic equilibrium

The previous section showed that general equilibrium is possible in a competitive market which consists of an arbitrary number of goods. Now let's discuss whether this mechanism is optimal in terms of pricing and resource allocation.

Consider optimality criterion of resource allocation, which was developed by economist V. Pareto. According to this criterion, resource allocation is effective if the situation of some economic agents cannot be improved without worsening the economic situation of others. If, when allocating resources, it is possible to improve the situation of some market participants without worsening that of others, such allocation is considered Pareto-preferred. There are three types of Pareto efficiency: distribution efficiency, production efficiency, and resource efficiency.

For better clarity, suppose that there are only two products: *X* and *Y*, which are produced in quantities X_n and Y_n. Next, assume that there are only two consumers: consumer *A* and consumer B. Let's describe the consumption efficiency using a special graph called an Edgeworth graph or Edgeworth box.

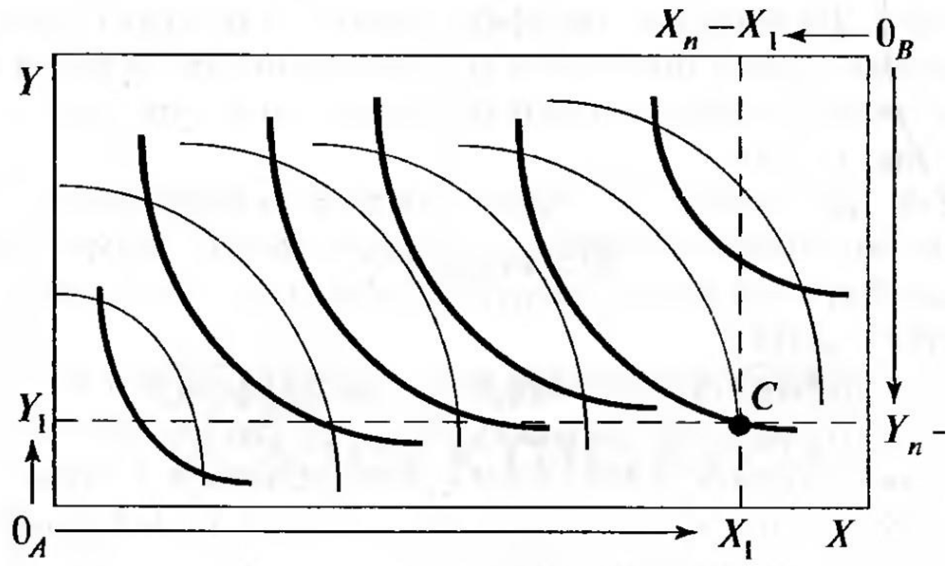


Fig.6.4. Edgeworth's Box

As can be seen from Figure 6.4, the Edgeworth box consists of two positive quadrants of Cartesian coordinate systems. One coordinate system is formed by two axes along which consumption quantities of goods *X* and *Y* by consumer *A* are plotted. The other coordinate system shows corresponding axes for consumer *B*. Each point on the Edgeworth graph refers to the distribution of goods *X* and *Y* between consumers *A* and *B*.

Suppose, consumer A's preference is described by a standard map of indifference curves which are convex to the origin. Consumer *B* has a similar utility function. Note that the indifference curves convex on the Edgeworth box with respect to consumer *A* are concave with respect to consumer *B*.

Let's choose point *C* in Figure 6.4. The distribution of goods at this point is characterized by non-optimality. This is due to the fact that two consumers' indifference curves which lie at this point do not touch each other. Let's find the set of points at which the indifference curves of both consumers touch. Now connect these points. The resulting curve is shown in Fig.6.5.

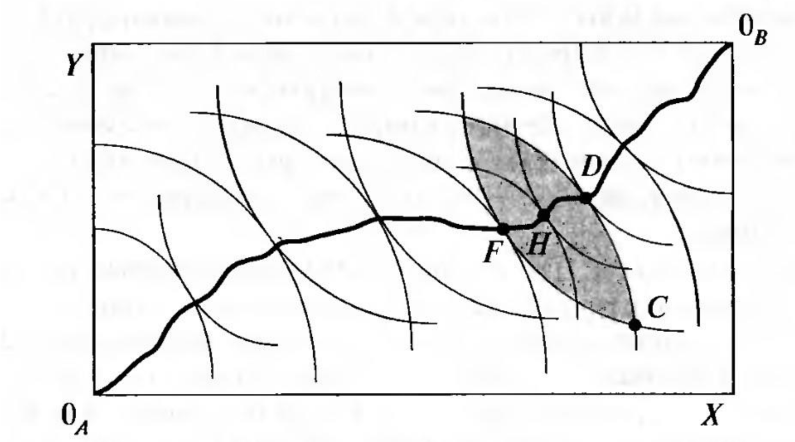


Fig.6.5. Contract curve

The line shown in Fig.6.5, is called ***contract curve***. Since the tangent angles at the points lying on this curve coincide, we can claim that in these the relation

$$MRS_{X,Y}^{A} = MRS_{X,Y}^{B}. \qquad (6.4)$$

When the ratio is fulfilled, Pareto efficiency of goods distribution is achieved. However, the question arises: at which point on the contract curve will an equilibrium distribution of goods between consumers *A* and *B* be established? The answer depends on the starting point of the goods distribution between the two consumers. Suppose, initially, the preference system was at point *C*. In this case, consumers will keep exchanging with each other until they reach a Pareto-efficient goods distribution. As a result, sooner or later they will come to a point lying on the contract curve. This point is called the ***core***. Each initial goods distribution has own core.

Thus, goods distribution efficiency in consumption can be found using the Edgeworth graph. Using the same technique, it is possible to find the efficiency of resource allocation between producers. Suppose that in some abstract economy there are only two producers: *A* and *B*, who produce two goods: *X* and *Y*. To produce these goods they use labor and capital.

Let's depict these manufacturers' isoquants on the Edgeworth box. To do this, we combine two Cartesian coordinate systems with axes showing the amount of labor and capital used. The resulting graph is shown in Fig.6.6.

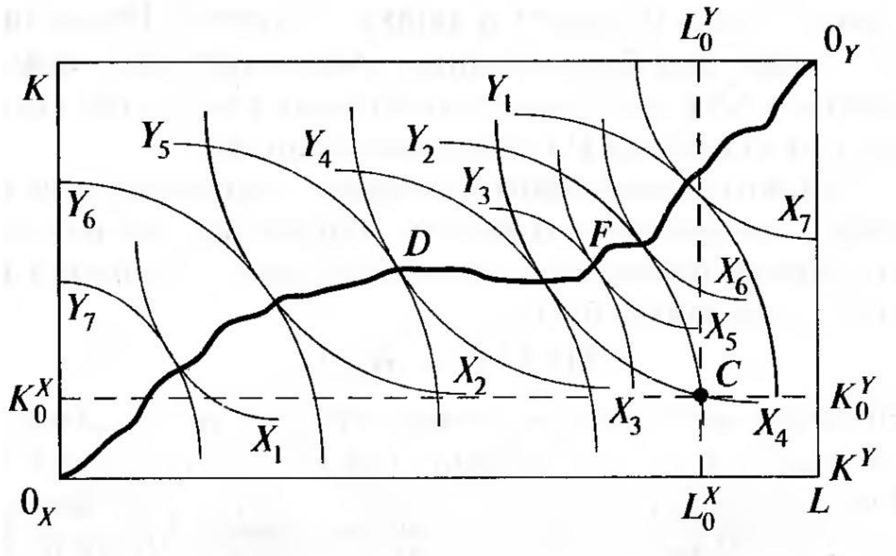


Fig.6.6. Production contract curve on the Edgeworth box

Figure 6.6 shows isoquants representing production functions for goods X and Y. Each point on the Edgeworth production box characterizes resource allocation between the two manufacturers. There is point C in Figure 6.5. At this point, resource allocation is not Pareto efficient because the isoquants at this point intersect at this point rather than touch each other.

Connecting all the points where the two manufacturers' isoquants touch each other with a line allows one to get a ***curve of production contracts***. This curve characterizes all variants of resource allocation which are Pareto-efficient. At these points, the following ratio is fulfilled

$$MRTS_{X,Y}^{A} = MRTS_{X,Y}^{B}. \tag{6.5}$$

Under the general equilibrium, Pareto efficiency is achieved in both production and consumption. However, efficiency in production can be achieved with a different distribution of factors of production and, consequently, with a different output structure. Production possibilities curve reflects Pareto-efficient production methods. A typical type of the production possibilities curve is shown in Fig.6.7.

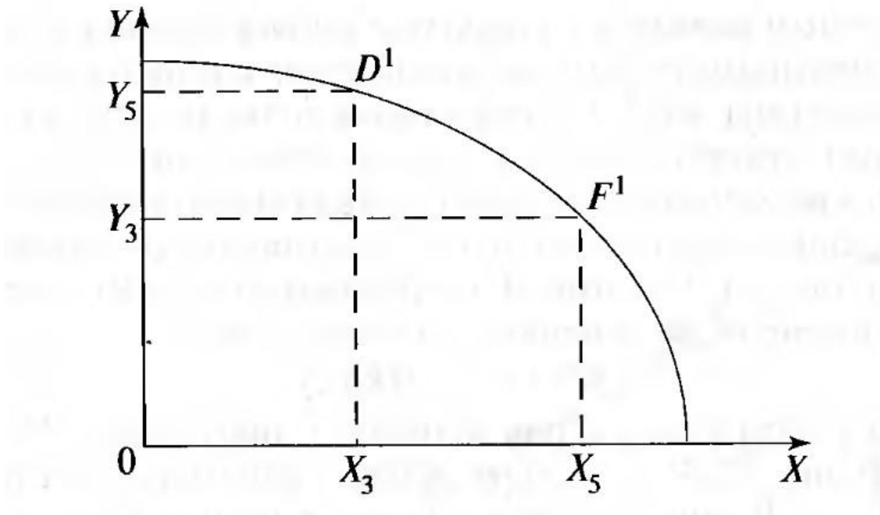


Fig.6.7. Production possibilities curve

Figure 6.7 shows a production possibilities curve that reflects Pareto-efficient output options. Each point on the production possibilities curve corresponds to a point on the production contract curve. However, the latter reflects the combination of labor and capital, at which optimal output is achieved. The production possibilities curve reflects the optimal production of goods *X* and *Y*.

An important characteristic of the production possibilities curve is the marginal transformation rate ($MRT_{X,Y}$). Geometrically, the marginal transformation norm at some point is equal to the tangent of the angle of inclination of the production possibilities curve in it. Algebraically, the marginal transformation rate is equal to the ratio of marginal costs and production of goods *X* and *Y:*

$$MRT_{X,Y} = \frac{MC_X}{MC_Y}. \qquad (6.6)$$

The Pareto-efficient output structure is achieved at the point where the marginal rate of transformation of goods *X* into goods *Y* is equal to the marginal rate of replacement of these goods:

$$\frac{MC_X}{MC_Y} = \frac{MU_X}{MU_Y}. \qquad (6.7)$$

For an economic system to be in the state of Pareto-efficiency, it is necessary to achieve Pareto-efficiency in production, consumption and output structure. This statement is called the ***first welfare theorem***.

The second welfare theorem states that if the condition of convexity to the origin of all isoquants and indifference curves is met, each Pareto-efficient distribution of goods and resources corresponds to a price system providing general equilibrium in all markets.

However, in practice, Pareto-efficient allocation of resources and goods is not always achieved. Possible reasons preventing the Pareto optimum to be achieved are commodity taxes and imperfection of competition. Suppose, a system has achieved Pareto-efficient equilibrium in which the following equality is satisfied

$$\frac{MC_X}{MC_Y} = \frac{P_X}{P_Y} = \frac{MU_X}{MU_Y}. \qquad (6.8)$$

Suppose the government has introduced a commodity tax on goods X in the amount of t rubles per unit of goods. In this case, when selling product X, sellers will be paid not $P_X$rubles, but $P_X - t$. So equality (6.8) is transformed into a relation$\frac{MC_X}{MC_Y} = \frac{P_X - t}{P_Y} = \frac{MU_X}{MU_Y}$. Consequently, the introduction of commodity taxes maintains efficiency in production and consumption, but does not provide the best output structure. The decision on the output structure depends on the initial prices, and the decision on the purchase – on the tax-adjusted prices.

Thus, it can be stated that when an excise tax or a commodity tax is introduced for product X, producers begin to shift the output structure in favor of other goods. In this case, the production of the specified good turns out to be understated. In turn, the production of subsidized goods gets overpriced.

Similarly, we can consider the impact a monopoly has on achieving efficiency in production and consumption. If product X is sold on a monopolized market, its production is understated in relation to the optimal one.

Example 6.2. In some abstract economy, only two goods are produced: X and Y. There are only two consumers in it: A and B with the same utility functions $U = XY$.. The equation of production possibilities curve in this economy is $X^2 + Y^2 = 5000$. Market prices for goods X and Y are equal; What are the outputs of products X and Y under the general equilibrium?

Solution. According to the formula (2.9), the general equilibrium is achieved at the point where equality $\frac{MC_X}{MC_Y} = \frac{P_X}{P_Y} = \frac{MU_X}{MU_Y}$ is satisfied.

Let's calculate marginal utilities $MU_X = Y;\ MU_Y = X$.

It follows from the condition that $P_X = P_Y$. Therefore$X = Y$. Let's substitute this equality into the equation of the production possibilities curve:

$2X^2 = 5000;$

$X^2 = 2500;$

$X = 50;$

$Y = 50.$

Answer. The optimal output of products X and Y is 50 units each.

As a result, a conclusion is drawn that any interference of a competitive market with free pricing results in non-optimal production. Some external effects in production and consumption also leads to a violation of Pareto- efficiency.

6.3 Theory of public welfare

In the previous section, it was found out that under Pareto-efficient distribution, the marginal replacement rate is equal to the ratio of market prices. However, the question arises if such an equilibrium is optimal in terms of maximizing the utility function of the whole society?

To answer this question let's turn to the Edgeworth box shown in Figure 6.5. The diagram shows Pareto-effective commodity sets that are formed as a result of the interaction of consumers *A* and *B*. Let's consider point *D*. Utility rates U_A and U_B received by individuals *A* and *B* correspond to this point . Next, coordinates $U_A - U_B$ hep to show all combinations of utility rates received by individuals *A* and *B* for which Pareto-efficient distribution of economic goods is achieved. Thus, we obtain a curve of possible utilities (Fig.6.8).

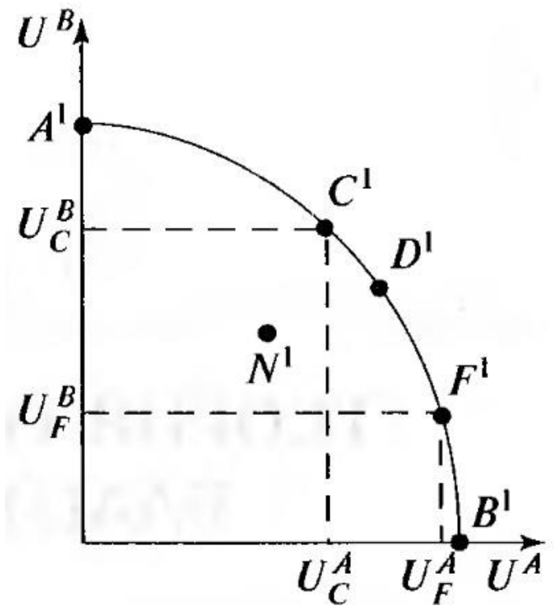


Fig.6.8. The curve of possible utilities

The intersection point of the curve of possible utilities and axis $U_B$corresponds to the situation when all public utility goes to individual *B*. Similarly, the point of intersection of the curve of possible utilities with the axis $U_A$corresponds to the situation when all public utility goes to individual *A*.

As can be seen from Fig.6.7, the curve of possible utilities is convex. In order to increase the utility of individual *A* under Pareto-efficient goods distribution, it is necessary to reduce the well-being of individual *B*. The tangent of inclination angle of the curve of possible utilities indicates the "alternative cost" of individual *A's utility*. It corresponds to the number of units of an individual *A*'s utility, which he should abandon in order to increase individual B's utility by one unit.

Each point on the curve of possible utilities reflects the total utility of all members of the society. Consequently, it is a function of public welfare and it reflects all possible utility rates received by all members of society when achieving Pareto-efficiency.

Suppose, point N^1 in Figure 6.7 indicates the state of the society. At this point, Pareto-efficiency is not achieved because the increase in the welfare of one individual is possible without reducing another individual's well-being. According to Pareto, optimality is only achieved at points located on the curve of possible utilities. But is the Pareto-efficient distribution optimal in terms of maximizing public welfare?

To answer this question, it is necessary to introduce the concept of public welfare function. Suppose that the same way as an individual prefers these or those goods, a society has some preferences concerning improving the well-being of its members. Let's indicate function of public welfare as *W*. It is a function of many variables where individual customers' utilities serve as arguments:

$$W = W(U_1, U_2, \dots, U_n). \tag{6.9}$$

Further, to simplify the analysis, let's assume that society consists of only two individuals – *A* and *B*. Thus, in coordinates $U_A - U_B$ it is possible to depict ***equal welfare curves*** indicating various combinations of the levels of attainability of individuals *A* and *B*'s utilities, welfare of the whole society remaining constant (Fig.6.9).

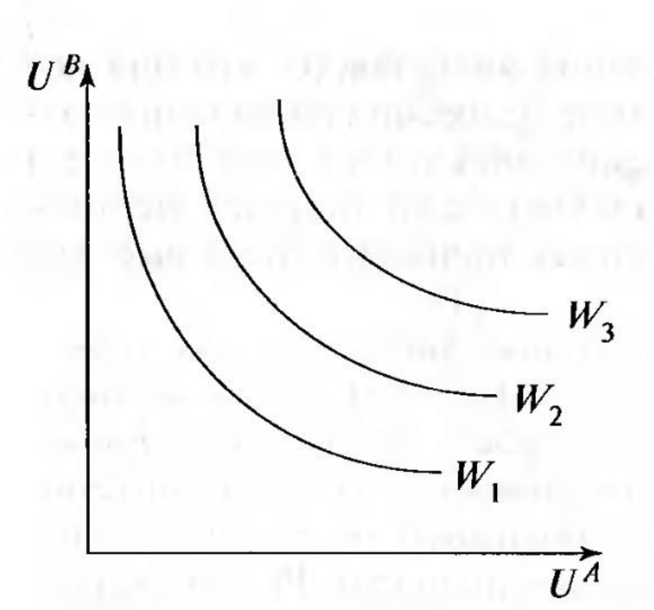


Fig.6.9. Equal welfare curves

Social welfare function can be represented as a map of equal welfare curves as well as particular individual's utility function can be represented by a map of indifference curves. Similarly to the utility function, the further the equal welfare curve, the greater the utility it brings to the whole society.

Let's connect the map of equal welfare curves with the curve of possible utilities (Figure 6.10). It is obvious that the highest need satisfaction rate for the whole society, in which p=Pareto efficiency is achieved, is possible at the point where the equal welfare curve touches the possible utility curve.

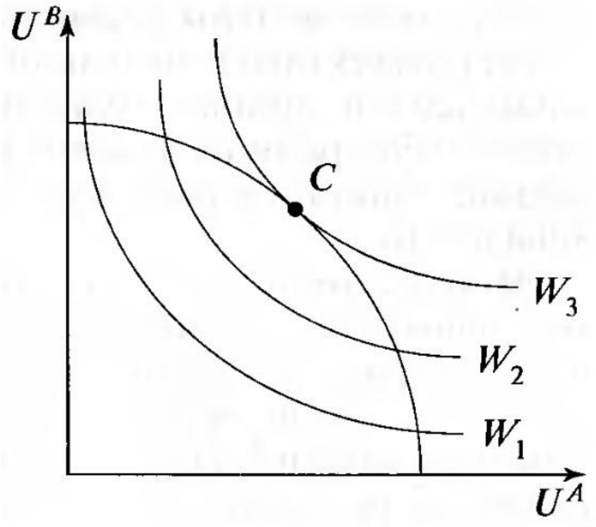


Fig.6.10. Public welfare maximizing

As can be seen from Fig.6.10, the public welfare maximum is achieved at point *C*. Analyzing the figure, a conclusion can be drawn that the maximum of public welfare is achieved when goods are distributed Pareto-efficiently. At the same time, not every Pareto-efficient distribution ensures maximum satisfaction of the needs of the whole society. Therefore, Pareto efficiency is a necessary but not sufficient condition for maximizing the public welfare function.

Let's discuss the type of public welfare function. Which individuals' needs are more valuable for society, and which individuals' needs are less valuable? There is no single point of view. However, there are three most well-known approaches to solving this problem: libertarian, utilitarian and egalitarian.

Libertarians believe that goods distribution, which is possible due to the functioning of competitive market mechanisms, is optimal for the society. The argument to support this approach is that it is only through the market mechanism functioning that the most efficient distribution of economic resources among producers and goods among consumers is achieved. With this approach, the equal welfare curve coincides with the possible utility curve.

One of the theorists of the libertarian approach was American philosopher R. Nozick. In his opinion, the distribution through the market mechanism is non-violent and purely voluntary. Consequently, it provides the most effective satisfaction of all the needs of the society. This approach does not clearly distinguish between the concepts of effective satisfaction of needs and equitable satisfaction of needs. Libertarians believe that the distribution resulting from the market mechanism is both effective and fair.

A significant drawback of the libertarian approach is that it does not take into account social inequality. The market mechanism does not take into account some circumstances. For example, some people inherit substantial capitals and have large financial resources, while not having intellectual or social abilities. It also does not take into account the fact that disabled people do not have opportunites to earn a living, and market distribution does not guarantee them a life in society.

The utilitarian approach was founded by English philosopher I. Bentham. He believed that the general welfare of a society was the sum of the welfare of all its members without drawing any difference between them. According to this approach, the public welfare function can be represented as

$$W(U_1, U_2, \dots, U_n) = \sum_{i=1}^{n} U_i. \tag{6.10}$$

In this case, the map of equal welfare curves can be described as straight lines having the same angle of inclination. Social optimum is reached at the point where the possible utility curve touches the function of public welfare (Fig.6.11).

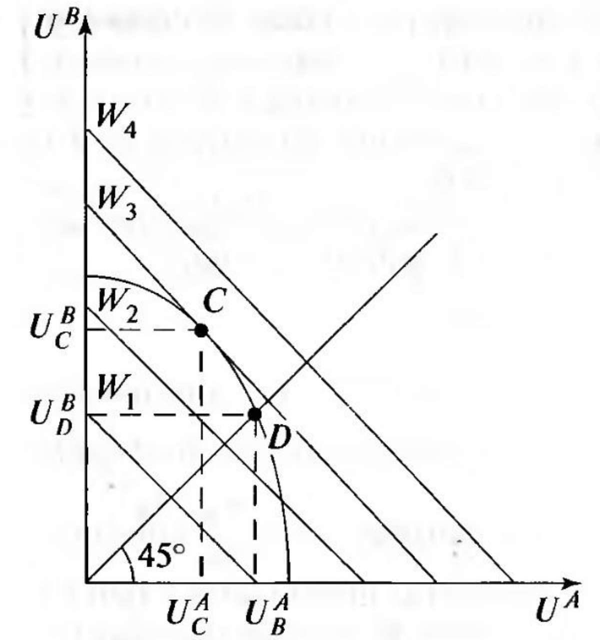


Fig.6.11. Maximization of public welfare under a utilitarian approach

Figure 6.11 shows that the public welfare maximum is achieved at point *C*. As can be seen from the graph, need satisfaction of individuals *A* and *B* is not achieved equally in this point. Consumer *A* gets more utility than consumer *B*. Meanwhile, the utilitarian approach pays more attention to the problem of social inequality than the libertarian one.

Developing this concept in the direction of increasing social equity, one can give different importance rates to utility functions of particular individuals:

$$W(U_1, U_2, \dots, U_n) = \sum_{i=1}^{n} \alpha_i U_i, \qquad (6.11)$$

where α_i *is* the importance that the state attaches to the *i*–th member of the society when conducting economic policy.

Typically, government bodies provide financial support to low-income citizens by redistributing financial resources from the rich to the poor.

According to the egalitarian approach, the concepts of social equity and the equality of income distribution among members of society are the same. When considering an extreme case of this concept, according to it, all capable citizens of the country should receive the same income. At the same time, it should be noted that the theory of public welfare states that it is not the income of the society that is subject to distribution, but its total utility. Further we will assume that the distribution of income and utility among the members of society are the same. A map of equal welfare curves under the egalitarian approach is shown in Figure 6.12.

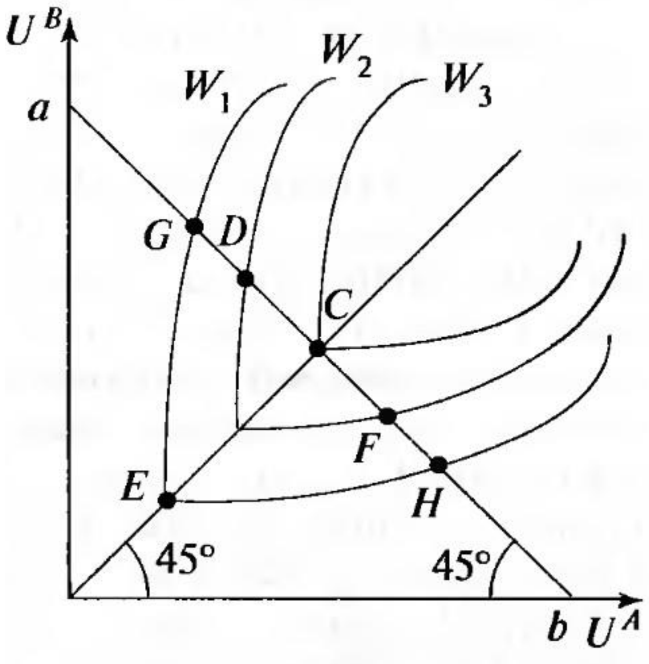


Fig.6.12. A map of equal welfare curves undder the egalitarian approach

Let's analyze the graph shown in Fig.6.11. The equal welfare curves shown in it are bending to a straight line which divides the positive quadrant in half. It can be explained by the following. Suppose the public utility is distributed between two individuals in the proportion corresponding to point *H*. In this case, one of the members of the society receives more utility than the other. That is why point *H* lies on a lower equal welfare curve than point *C*, which corresponds to an equal distribution of goods among all members of the society.

However, there are currently no countries whose goal is an absolutely equal distribution of income among all their citizens. Most countries only try to level inequality in income distribution resulting from market mechanism functioning. In this case , the public welfare function can be algebraically represented as a Cobb – Douglas function of the utility of its individual members:

$$W(U_1, U_2, ..., U_n) = \prod_{i=1}^{n} U_i^{\alpha_i} \quad (6.12)$$

So, there are several approaches to assessing public welfare through the utility functions of individuals. Here another question arises on which of these approaches is the best. In the early 1950s American economist K. Arrow proved a theorem according to which it is impossible to obtain from individual utilities an aggregated public welfare function that would satisfy all the the utility function axioms. In addition, he assumed that the society does not have a dictator who would satisfy the needs of only a certain part of the country's population.

Example 6.3. The curve of equal utilities of individuals A and B is given by the ratio $U_A + U_B^2 = 300$. The public welfare function has the form $W(U_A, U_B) = U_A U_B$. Find a utility distribution that is both Pareto-optimal and maximizes the public welfare.

Solution. In this problem, it is necessary to maximize the function of public welfare, provided that the maximum is found on the equal utility curve. This can be done by maximizing the Lagrange function: $L(U_A, U_B, \lambda) = U_A U_B - \lambda(U_A + U_B^2 - 300) \rightarrow max$.Next, we should equate the partial derivatives to zero:

$$\begin{cases} \dfrac{\partial L}{\partial U_A} = U_B - \lambda = 0 \\ \dfrac{\partial L}{\partial U_B} = U_A - 2\lambda U_B = 0 \\ \dfrac{\partial L}{\partial \lambda} = 300 - U_A - U_B^2 = 0 \end{cases} \rightarrow \begin{cases} U_B = \lambda \\ U_A = 2\lambda U_B \\ U_A + U_B^2 = 300 \end{cases} \rightarrow U_A = 300 - \lambda^2 = 2\lambda^2 \rightarrow \lambda^2 = 300 \rightarrow$$

$$\rightarrow \lambda = \sqrt{300} \rightarrow U_A = 0 \rightarrow U_B = \sqrt{300}.$$

Answer. The optimum of the public welfare function is achieved when the utility of individual A is zero and the $\sqrt{300}$utility of individual B is the same.

6.4 Externalities

In the previous section, it was shown that general economic equilibrium is only achieved under completely competitive markets. Consequently, an economic system comes to equilibrium only as a result of market mechanisms. Nevertheless, in modern microeconomics there are five reasons why market mechanisms do not lead to a general equilibrium in the economic system. These reasons are: the existence of monopolies, uncertainty and risk, externalities, public goods and information asymmetry. In this section, the first type of so-called market failures will be considered: external effects.

Externalities are the costs or benefits of a transaction received by a third party and not taken into account by the parties to the agreement at the time of its conclusion. There are external effects in consumption and external effects in production. Here is an example of an external effect in consumption. Student Ivanov bought an audio recorder with powerful speakers. However, loud music prevents neighbors from sleeping. This is an example of an external effect in consumption. An example of an external effect in production is air pollution. Suppose, a financial holding company is panning to build a factory in city *X*. However, it will cause air pollution for the residents of this city. Thus, in this example, air pollution is an example of a negative external effect in production.

In neoclassical microeconomics, externalities are examples of so-called market failures, that is, situations when the market of perfect competition does not provide Pareto-efficient distribution of goods and resources.

Negative externalities in production are associated with external marginal costs indicated by *MD*. According to this concept, there are social marginal costs (*SMC*) and private marginal costs (*PMC*). Private marginal costs are the sum of the marginal costs of all firms in a given industry. As for public marginal costs, these are the sum of private marginal costs and external marginal costs:

$$SMC = PMC + MD. \tag{6.13}$$

SMB is the function of market demand. Let's graphically depict the market equilibrium in the presence of externalities (Fig.6.13).

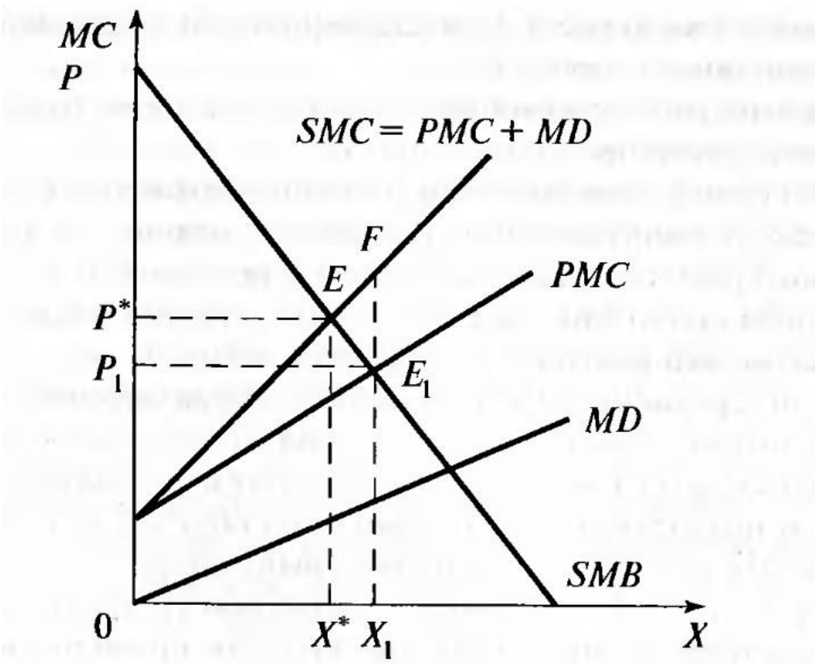


Fig.6.13. Market equilibrium in the presence of negative externalities

As can be seen from Fig.6.13, under the perfect competition in the market, the sales established are X_1 and price established is P_1. However, in terms of Pareto-efficient production, the equilibrium should be established at the point where $SMC = SMB$. From this a concluded can be drawn that in case of externalities, market equilibrium is not Pareto-efficient. Another conclusion: if the externalities are negative, the market gives overvalued sales and an undervalued price. if the externalities are positive, the prices are overvalued and the sales are undervalued.

Everything concerning negative externalities can be extended by analogy to positive externalities. In this case, instead of the curve of external marginal costs, there is a curve of external marginal benefits, which is indicated as *MB*. Meanwhile, the interaction of firms at the perfect competition market results in overvalued prices and undervalued sales.

The state policy should be aimed at minimizing negative consequences of both positive and negative externalities. For example, the Russian Federation introduces so-called environmental taxes for industries that cause environmental pollution.

Modern scientific and technological progress has led to a wide spread of markets of ***network products***. A distinctive feature of network products is that their utility increases as they become more widespread. A situation when the value of a product increases when the number of people who use this product increases too is called ***network effect***.

Mobile communications or social networks are examples of network products. The more subscribers are connected to a mobile operator, the higher the probability that an individual subscriber will find his friends on the mobile network. Network effects are also typical for complementary goods markets. For example, there is no point in selling printers in the place where there are no personal computers. The advent of information technology gave a fresh impetus to the development of network effects.

In this regard, network markets often move towards monopolization. For example, there is no point in using a social network consisting of one hundred users. However, a network monopoly has its own peculiarities compared to a conventional monopoly. Unlike a conventional monopoly, a network monopoly does not take all the consumer's surplus to itself. Nevertheless, the emergence of network monopolies makes antimonopoly regulation more difficult. This is due to the fact that in a network product market, it is not necessarily the seller that most satisfies the consumers' needs that becomes a monopoly. It may be a seller of a popular software product, which at one time received considerable funding from a major investor.

Thanks to network effects, a monopoly becomes very stable. For example, some startup has developed a new social network that better meets the needs of consumers than the already well-known social networks. However, it cannot gain much popularity among users due to the fact that the range of potential subscribers of the new social network is very limited.

In addition, a network monopolist firm may have advantages in terms of the production of complementary goods. The production of MS Office software package, which can only work with a limited set of operating systems, can illustrate the point. Such effects are often realized within a framework of a cartel agreement.

The American economist Arthur Cecil Pigou proposed to adjust externalities by taxation. According to Pigou's approach, the government should establish a pollution tax in the amount equal to the public costs of these pollutants.

Pigou's approach is theoretically perfect but in terms of practical implementation it has a few difficulties. This is due to the fact that in practice, state regulatory authorities often do not know the amount of damage that pollution causes to society.

According to another approach, the state establishes a certain acceptable level of pollution for a particular company. All emissions into the atmosphere above this level are to be taxed.

The third approach to environmental taxation involves creating a special market for licenses for polluting production. According to this approach, the state sells a quota for the release of pollutants into the atmosphere, water and soil.

Also, there is the fourth approach to taxation of externalities. According to this concept, the State introduces administrative responsibility for negative externalities. The individual suffering from the negative consequences of a certain product can apply to the court with a claim for compensation for the harm that this economic good causes them.

For a long time, the Pigou's approach was considered an ideal theoretical concept for taxes on negative externalities. However, in 1960 R. Coase published an article where he put forward the idea that Pigou's theory had a few drawbacks.

Meanwhile, R. Coase focused on the concept of ***transaction costs***. He considered transaction costs as the costs of attracting buyers and drafting purchase and sale contracts. According to Coase, the price of harmful goods must be lowered not by taxation, but by reducing their public harm. For example, a factory, whose production pollutes water, has to install a special cleaner.

Meanwhile, it should be noted that Coase's argumentation was based on the assumption that society has zero transaction costs. If it is so, all the above arguments are not quite correct.

A special place in the theory of market failures belongs public goods. They have the following characteristics:

1. Additional consumption of these goods does not reduce their rarity.
2. It is impossible to prevent the consumption of these goods by those who do not want to pay for these products.

Examples of public goods include non-rush-hour highways, land, pastures, access to cable television, street lighting, and national defense. Further, it can be shown that the availability of public goods leads the economic system to a state that is not Pareto-efficient. This is due to the fact that public goods bring different utility for different individuals. Consequently, different individuals may pay differently for the use of this product or service. That is why, the mechanisms of perfect market competition, bringing positive external effects, cannot ensure the optimal distribution of public goods. Therefore, such a distribution is not Pareto-efficient.

This chapter revealed that in the case when the economic system consists exclusively of perfect competition markets, the general equilibrium is possible. At the same time, monopolies and commodity taxes can lead to the establishment of a stable state in the system, which would be different from the equilibrium.

In addition, under the conditions of perfect market mechanisms, the equilibrium into which an economic system consisting of various markets comes, is Pareto-optimal. At the same time, the science has not studied the question of whether a Pareto-efficient equilibrium is optimal for society in terms of meeting its needs. Supporters of the libertarian approach believe that market mechanisms are able to ensure the most efficient distribution of income among the citizens of the country. At the same time, most economists believe that the state should redistribute income between market participants to support the least socially protected segments of the population.

TASKS FOR SELF-SOLVING FOR CHAPTER 6

1. Consider the interaction of markets of two products *X* and *Y*. The demand functions for them are given by relations $Q_X^D = 20 - P_X - 0,5P_Y; Q_Y^D = 30 - P_Y + 0,25P_X$. The supply functions are given by ratios: $Q_X^S = 10 + P_X + 0,4P_Y$; $Q_Y^S = 20 + P_Y + 0,6P_X$. Determine the prices and sales volumes at which equilibrium is established in both markets.

2. In some abstract economy, only two goods *X* and *Y* are produced. There are only two consumers: *A* and *B*, they have the same utility functions: $U = XY$.. The equation of the curve of production possibilities in this economy is $X^2 + Y^2 = 5000$. Market prices for goods *X* and *Y* are equal. The price of good *X* is 2 rubles, the price of good *Y* is 4 rubles. Is the general equilibrium achievable with this price ratio?

3. The economy uses two resources: labor (*L*) and capital (*K*), and produces two goods *X* and *Y*, which are consumed by two individuals *A* and *B*. The production functions of both goods are the same and are determined by ratios $X = K^{1/4}L^{3/4}$; $Y = K^{1/4}L^{3/4}$. Both consumers have the same utility functions: $U = X^{1/4}Y^{1/4}$. Under the general equilibrium, 16 units of capital and 256 units of labor are used to produce good *X*; 1296 units of labor and 81 units of capital are used to produce good *Y*. 25 % of the

total amount of goods X and Y are made available to individual A. Determine equilibrium price ratio for goods X and Y($\frac{P_X}{P_Y}$).

4. The father of the family bequeaths 1,800 sheep to his two sons. The first son's utility function is $U_A = X^{1/2}$, the second son's utility function is $U_B = 2X^{1/2}$ (X is the number of sheep). The utility function of the family welfare has the form: $W(U_A, U_B) = U_A^{1/2} U_B^{1/2}$. How should the sheep be divided?

SELF-CHECK TEST FOR CHAPTER 6

1. Suppose that the gasoline price increases due to a decrease in supply. What will happen to car prices and car sales volume according to the theory of general economic equilibrium?

a) the price will increase, the sales volume will decrease;
b) both the price and sales volume will decrease;
c) both the price and sales volume will increase;
d) the price will decrease, the sales volume will increase.

2. The chewing gum market is dominated by two products: "Dirol" and "Orbit". As a result of a successful advertising campaign, the demand for chewing gums "Orbit" has grown, the demand in the market as a whole remaining the same. What will happen to the price and sales volume of chewing gum "Dirol"?

a) the price will increase, the sales volume will decrease;
b) both the price and sales volume will decrease;
c) both the price and sales volume will increase;
d) the price will decrease, the sales volume will increase.

3. Chose a utilitarian function from the following types of a public welfare function:

a) $W(u_1, u_2, ..., u_n) = \sum_{i=1}^{n} u_i$;
b) $W(u_1, u_2, ..., u_n) = \max\{u_1, u_2, .., u_n\}$;
c) $W(u_1, u_2, ..., u_n) = \min\{u_1, u_2, .., u_n\}$;
d) all of the above.

4. If, as a result of the redistribution of benefits, individual A has received more than individual B has lost, it means that the initial distribution:

a) is Pareto-efficient;
b) is Pareto-inefficient;
c) the final Pareto state is preferable to the initial state;
d) there is no right answer.

5. Before the introduction of the excise tax, the system was Pareto-efficient in terms of production, consumption and distribution of goods. After the introduction of the excise tax, Pareto-efficiency may be violated:

a) in the consumption;
b) in the distribution;
c) in the production;
d) both b and c are true.

6. Which of the philosophers believed that the income distribution which is established as a result of the market mechanisms is the most optimal:

a) Jeremiah Bentham;
b) Friedrich Nietzsche;
c) Kenneth Arrow;
d) all listed.

7. If there is a positive external effect, the economic good is produced:

a) in a larger volume than the socially efficient volume of output;
b) in a larger volume than the socially effective volume of output under a monopoly;
c) in a smaller volume than the socially efficient volume of output;
d) there is no right answer.

8. If there is a nagative external effect, the economic good is produced:

a) in a larger volume than the socially efficient volume of output;
b) in a larger volume than the socially effective volume of output under a monopoly;
c) in a smaller volume than the socially efficient volume of output.
d) there is no right answer.

PREPARATION FOR THE FINAL TESTING

The final test consists of a test including 15 theoretical questions and four problems aimed at the assessment of practical skills of students. The test is estimated at 15 points. The maximum score for the correct solution of problems is 15 points. Below is a demo version of the final test.

The theoretical part

Each correct answer is estimated at 1 point

1. What are the two most typical features distinguishing a system from a group of random objects:
   a) the system has stable connections between the elements;
   b) the system can accept new elements;
   c) all elements of the system have a common goal;
   d) the system does not always have a material representation.
2. Can a subsystem be divided into elements?
   a) yes, because a subsystem is a divisible part of the system;
   b) no, because a subsystem is an indivisible part of the system;
   c) yes, because any system has a hierarchy of system – subsystem – element;
   d) no, because a system can consist of subsystems, but it cannot consist of elements.
3. The goal-setting law states:
   a) the goal of the system development is determined by objective natural and social laws;
   b) each system must have a major goal;
   c) the system must have a hierarchy of goals;
   d) the goal of the system should be clearly stated.
4. Simultaneous increase in both demand and supply will lead to:
   a) both price and sales increase,
   b) falling prices and increasing sales;
   c) falling prices and sales volume;
   d) price and sales effects will depend on the slope of the supply and demand curves.
5. An indifference curve shows:
   a) all commodity sets bringing similar utility to the consumer;
   b) the ratios of goods *X* and *Y* in which the saturation point is reached;
   c) the ratio of labor and capital at which the maximum level of utility is reached;
   d) the ratios of goods *X* and *Y* at which incomes are equal.

6. When the consumer's income increases, the budget constraint line:
   a) will shift parallel up;
   b) will shift parallel down;
   c) will change its slope;
   d) will shift up, but not parallel.
7. With an increase in income the demand for low-value goods:
   a) falls;
   b) increases;
   c) increases faster than income;
   d) can either increase or decrease.
8. Let there be product *X*. Its income effect is greater than its substitution effect and acts in the opposite direction. It means that product *X* is:
   a) a normal product;
   b) a low-value product;
   c) a luxury good;
   d) a Giffen good.
9. The production function $y = f(K, L)$ is characterized by positive returns to scale. The amount of labor and capital used increases by 20%. In this case the output:
   a) will increase by more than 20 %;
   b) it will increase in the range between 15 and 20 %;
   c) will remain unchanged;
   d) it will increase by exactly 20%.
10. A larger output is represented by isoquants that lie relative to the original one:
    a) above and to the right;
    b) below and to the right;
    c) below and to the left;
    d) above and to the left.
11. In firm *A*, the average labor product is 10 with the number of permanent employees 15. In this case, the output of firm *A* is equal to:
    a) 150;
    b) 1,5;
    c) 200;
    d) 300.
12. Plant *X* bought a production machine for 20 000 rubles and an office desk for the plant manager for 10 000 rubles. In this case, the variable costs will be:
    a) 30 000 rubles;
    b) 10 000 rubles;
    c) 20 000 rubles;
    d) 50 000 rubles.

13. Let that there is a price in the market that is less than the average variable costs of firm *A*. In this case, the firm:
    a) makes a positive profit and stays on the market;
    b) makes a negative profit and remains on the market;
    c) makes a negative profit and leaves the market;
    d) makes a positive profit and leaves the market.
14. In which case does the firm appropriate the entire consumer surplus?
    a) in a market of perfect competition;
    b) in a monopoly market with the first-degree price discrimination;
    c) in a monopoly market with second-degree price discrimination;
    d) in a monopoly market with the third-degree price discrimination.
15. Which of the following markets is likely to be an oligopoly?
    a) of aircrafts;
    b) of oil;
    c) of cars;
    d) of educational services.

Practice

1. Citizen Stepanov's income is 100 rubles. The consumer set consists of bread and milk. The price of bread is 5 rubles per kilogram, that of milk is 10 rubles per liter. Stepanov's utility function is $U = X^{1/4}Y^{1/2}$, where *X* is the amount of bread consumed, *Y* is the amount of milk consumed. What commodity set is optimal for Stepanov? *(4 points)*
2. The production function of a firm is as follows $Q = 2\sqrt{KL}$. The pay rate is 4 rubles, the capital expenditure rate is 3 rubles. The company has monetary resources in the amount of 25 rubles. What combination of labor and capital is optimal for the company? *(4 points)*
3. The production function of a firm is as follows $Q = 4K^{1/4}L^{1/2}$. The pay rate is 10 rubles. The capital expenditure rate is 20 rubles. What is the firm's cost of producing 30 production units if it seeks to maximize its output? *(4 points)*
4. The company's cost function is: $TC = \frac{Q^3}{2} - 4Q^2 + 10Q$. At what output is the minimum average cost achievable? *(3 points)*

LIST OF RECOMMENDED LITERATURE

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